CHAPTER 3
MEASURING STARS

3.1 Introduction

The Sun, we have asserted, is a star. But all of the other stars are so far away that they appear as mere points of light in the sky, seemingly unchanging. And yet there have always been clues that the starry heavens are not as changeless as they appear to be. On rare occasions a new star will flare up in the sky, perhaps becoming visible in daylight, before fading in a few months back into invisibility. Such bright new stars are rare, and before the advent of the telescope in 1609, only ten or so had been recorded. Of particular importance was the one observed by the Danish astronomer Tycho Brahe ('Tie-co Bra-hay') (1546–1601) in 1572. Using the method of trigonometric parallax, which you will meet shortly, he was able to show that the new star lay beyond the Moon, thus overthrowing the then prevailing view that transient phenomena were confined to the volume of space closer to us than our satellite. So here was the first observational evidence that all regions of the cosmos are subject to change.

We now know that Tycho's new star was a *supernova*, which, as you will see in Chapter 8, is a massive star ending its life in a gigantic explosion. This has been established through painstaking studies of the radiation from those points of light in the sky. Massive stars are rare, and evolve quickly, particularly at the supernova stage. Earlier stages of their evolution, and the evolution of less massive stars, are usually too slow to be observed directly. We thus have to piece the evolution story together from observations of many stars of each particular type, at as many stages in each type's evolution as we can observe.

The observable properties are, however, almost entirely of *external* appearances and *external* events, and can take us only so far in revealing the evolution of the stars. To go further we must understand their interiors, and in Chapters 6 and 7 we shall be particularly concerned with how this understanding has been developed, and where it leads us in our understanding of stellar evolution. In this chapter, however, we shall be concerned with the essential preliminary step, namely measuring the observable properties of the stars.

The key properties that we need to characterize stars sometimes require units which are unfamiliar to us in everyday life or other branches of science. For example, the distances between stars are so vast that familiar length units such as kilometres are totally inadequate. Astronomical measurement systems and nomenclature have evolved over time as scientists' understanding and the technology have become more refined. In some cases this has resulted in an apparently arbitrary nomenclature, which is often confusing at first. However, such systems have been retained even when the basis of the system has been modified as a result of a greater understanding of the physical processes involved.

But before we look at the stars as bodies, what about the stars in space: how much further away than the Sun are the stars? Are the constellations really fixed, and how are stars distributed across the sky?

3.2 Stars in space

3.2.1 Are the stars fixed in space?

For thousands of years, and perhaps ever since the human species emerged on this planet, we have identified patterns in the stars – the constellations – and given them names. Today, we still use constellation names that originated in antiquity. Figure 3.1 shows the relative positions of the brighter stars in one of the better known constellations, that representing the mythological hunter Orion.
The boundaries of the 88 constellations used today have been defined by the International Astronomical Union, the governing body of astronomical names. The brightest stars have proper names, of Greek, Roman or Arabic origin, but generally stars are referred to by letters or numbers. The brightest stars in a constellation are referred to by Greek letters (usually, but not always) in order of brightness followed by a constellation designation, often shortened to three letters. For example, Betelgeuse (‘betel-jers’) is α (alpha) Orionis, or α Ori, and Rigel (‘rye-jel’) is β (beta) Ori. The Greek alphabet and constellation abbreviations are listed in Appendix A2. When a photograph of the constellation is taken, many more stars, invisible to the unaided eye, are seen. Many of these are referred to by their designations in star catalogues but most fainter ones have no names at all.

How fixed are these patterns of stars? Let’s take perhaps the best known of all, the Plough (known in North America as the Big Dipper), as an example. Figure 3.2 shows the Plough as it appears now and as it appeared to our ancestors 100 000 years ago, during the Old Stone Age. It certainly looks different. However, over a human lifespan the change is negligible. Figure 3.2c shows the Plough about 70 lifespans, or 5000 years into the past, at the beginning of written history.

Figure 3.1 (left) The patterns of the brighter stars in Orion. (right) A photograph of the same region taken in visible light showing the range of colours of stars (see Sections 3.3.2 and 4.2). The Orion Nebula, visible in this image as a reddish glow, is a dense cloud of gas and dust within which stars are forming, and is visible to the unaided eye. (Photo: D. Malin/AAO)

■ Compare this with the Plough today, in Figure 3.2a. Do you think that, to the unaided eye, it would have been noticeably different 5000 years ago?

❑ No, though it would have been easier to find 5000 years ago, when light pollution was a lot less!

Not surprisingly, it was a long time before such stellar motions were discovered, in 1718 by the British astronomer Edmund Halley (1656–1742). The motions continue, and Figure 3.2d shows the Plough as it will appear 100 000 years in the future.

The motion of a star across the sky is called its **proper motion** – so-called because it is intrinsic to the star and not a result of the motion of the observer or a moving reference point. It is usually expressed in seconds of arc per year, arcsec yr^{-1} (3600 arcsec = 1 degree).

■ What further information would need to be added to Figure 3.2 to enable you to calculate the proper motions?

❑ A fixed reference point is needed, with zero proper motion. (In fact, *all* the stars shown happen to have non-zero proper motions, and all the values are around 0.1 arcsec yr^{-1}, but in different directions across the sky.)

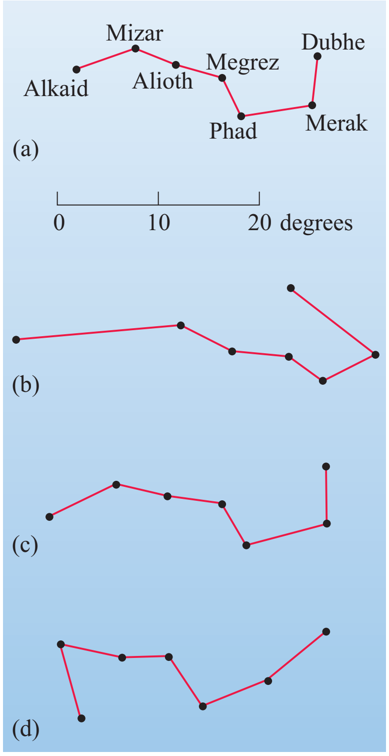

Figure 3.2 (a) The Plough; part of a larger constellation called the Great Bear (Ursa Major) as it appears today, and (b) 100 000 years ago, (c) 5000 years ago, and (d) 100 000 years in the future.

Currently, the proper motion record is held by Barnard's Star, a faint star in the constellation Ophiuchus with a proper motion of 10.4 arcsec yr^{-1}. By contrast, Rigel has a minuscule proper motion of less than 0.002 arcsec yr^{-1}.The proper motion of a star arises from its motion relative to us, in particular the component of its motion in a direction transverse to our line of sight to the star, as shown in Figure 3.3. This is called the **transverse velocity**. The magnitude of the transverse velocity, the transverse speed v_t, is given by

$$v_t = d \times \sin \mu$$

where μ is the proper motion and d is the distance to the star. Since μ is a very small angle, we can approximate $\sin \mu$ by μ measured in radians (per unit time), so

$$v_t = d \times (\mu/\text{radians}) \qquad (3.1)$$

Thus, a small proper motion at a large distance corresponds to a larger transverse speed than the same proper motion at a small distance. You will learn about the measurement of stellar distances later, but v_t varies enormously, from under 1 km s^{-1}

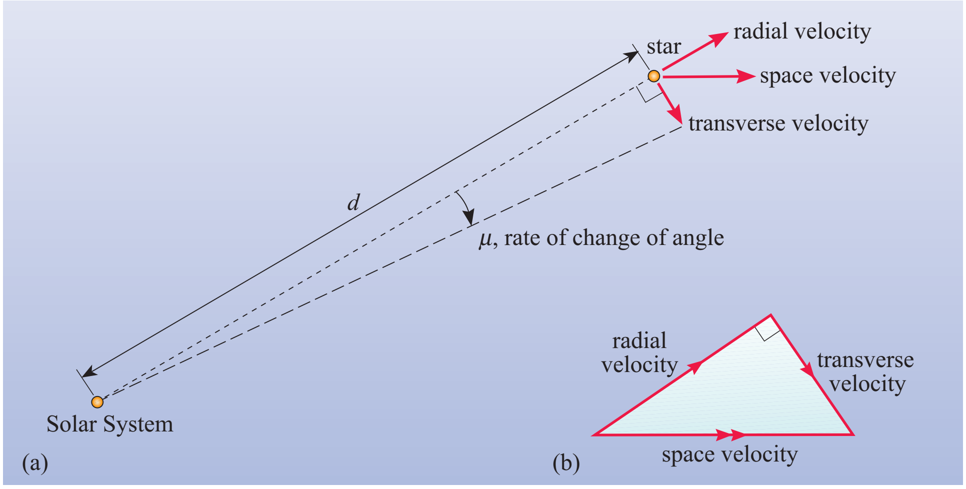

Figure 3.3 (a) A star's motion through space, relative to the Sun. (b) The overall velocity through space, the space velocity, has two components: the radial velocity in the observer's line of sight and the transverse velocity in the plane of the sky. (Space velocity will be described below.)

to well over 100 km s^{-1}. (For Barnard's star and Rigel, the values are 89 km s^{-1} and 2.2 km s^{-1}, respectively.)

Earlier, we gave μ in arcsec yr^{-1}, and now, via Equation 3.1, we have v_t in km s^{-1}. Let's follow this through. With μ in arcsec yr^{-1} and d in km, Equation 3.1 gives v_t in

$$\mathrm{km}\left(\frac{\mathrm{arcsec\,yr^{-1}}}{\mathrm{radians}}\right) \quad \text{or} \quad \left(\frac{\mathrm{km}}{\mathrm{yr}}\right)\left(\frac{\mathrm{arcsec}}{\mathrm{radians}}\right)$$

This is the same as $(\mathrm{km\,s^{-1}})\left(\dfrac{\mathrm{s}}{\mathrm{yr}}\right)\left(\dfrac{\mathrm{arcsec}}{\mathrm{radians}}\right)$

Note that (s/yr) is the number of years in a second ($1/(3.16 \times 10^7)$) and (arcsec/radians) is the number of radians in a second of arc (1/206 265). Therefore (s/yr) and (arcsec/radians) are both pure numbers, and so v_t is in units of km s^{-1}.

The component of the star's motion relative to us is called its **radial velocity**. In Figure 3.3 it is directed away from the Earth, but it could equally well have been directed towards us. Unlike the transverse velocity, the radial velocity can be obtained without knowing the distance to the star. The method relies on the Doppler effect (named after Christian Doppler, Figure 3.4), which has many applications in astronomy. The Doppler effect is described in Box 3.1.

If a star behaves in any way like a car horn, then we can use the Doppler effect to obtain the radial velocity. A star does indeed have the equivalent of a horn – in the radiation that it emits. In Chapter 1 you learned that the solar spectrum exhibits many spectral lines. This is also the case for any stellar spectrum. A spectral line is like a car horn in that it corresponds to an emission or an absorption at a specific frequency. Moreover, we can identify the atomic transition giving rise to a spectral line, and thus we know the emitted frequency ('emitted' here covers absorption lines as well as emission lines). Therefore, if the frequency we observe differs from the emitted frequency, then from the size of the difference we can use Equation 3.3 (with the speed of light, c, in place of c_s) to obtain the radial speed, and from the

CHRISTIAN ANDREAS DOPPLER (1803–1853)

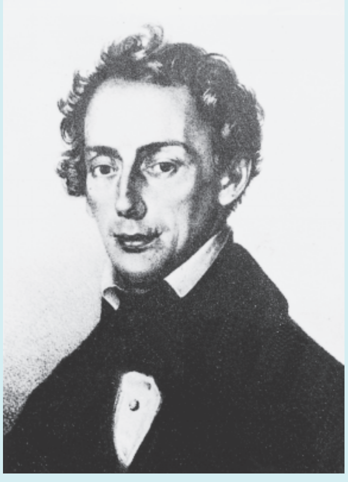

Figure 3.4 Christian Doppler. (Science Photo Library)

Christian Doppler (Figure 3.4) was born into a family of successful stonemasons in Salzburg, but was unable to take over the business due to poor health. His scientific abilities led him to the study of higher mathematics, mechanics and astronomy at the University of Vienna where he subsequently gained a post as assistant to the professor of higher mathematics. The difficulties of obtaining a permanent academic post in Austria at the time (which involved a centralized competition, with written and oral examinations and centred on teaching rather than research abilities) led him to consider emigration to America before he finally obtained a post in Prague. His research activities were hampered by onerous teaching and examination duties (for example, in July 1847 he orally examined over 800 students) and complaints from students that his examining was too harsh. His work on the Doppler effect was clearly the high point of his career. Despite regarding light as longitudinal rather than transverse waves and erroneously using his theory to explain the colours of double stars, he predicted the future importance of the effect in determining the motions of stars. Although it was not possible to observe the effect on light at the time, experiments involving musicians playing instruments on approaching and receding trains confirmed it.

BOX 3.1 THE DOPPLER EFFECT

The **Doppler effect**, named after the Austrian physicist Christian Andreas Doppler is the name given to the observed change in frequency of the waves emitted by a source when it is moving with respect to the observer. It is familiar in the change in pitch of the sound received from a car horn as the car sweeps past. As the car approaches, the pitch is higher – the frequency is higher; as the car recedes, the pitch is lower – the frequency is lower.

Figure 3.5 shows how the Doppler effect arises. In Figure 3.5a the car is stationary with respect to the observers at A, B and C. Its horn emits sound at a frequency f, and the sound waves spread out at the speed of sound, c_s, in the air. The circles are separated by one wavelength, λ, of the sound, given by

$$\lambda = c_s/f \qquad (3.2)$$

The observers at A, B and C all hear the same frequency, f, as that emitted by the horn. There is no Doppler effect.

In Figure 3.5b the car is moving with respect to the observers, and again it emits sound at a frequency f. Once the sound is emitted it still travels away from the car through the air at the speed c_s, which is unchanged by the car's motion. Thus the motion of the car causes the waves to pile up ahead of it – since each successive wave is centred on a new position of the car – giving rise to a decrease in wavelength at A, and hence (Equation 3.2) to an increase in frequency. Behind the car the waves are spread apart, giving rise to an increase in wavelength at B, and hence to a decrease in frequency. When the motion is perpendicular to the line from the car to the observer then there is no change in frequency: the observer at C hears the emitted frequency. The change in frequency thus requires that the velocity of the car has a component along the direction from the car to the observer, that is it requires a radial velocity; a transverse velocity produces no Doppler effect. It can be shown that the radial velocity is given by

$$v_r = c_s \times (f - f')/f' \qquad (3.3)$$

where f' is the observed frequency. Thus, the Doppler effect provides us with a way to measure radial velocities. (A negative value of v_r results from a motion towards the observer, as at A, i.e. in a direction opposite to the radial direction of the source from that observer).

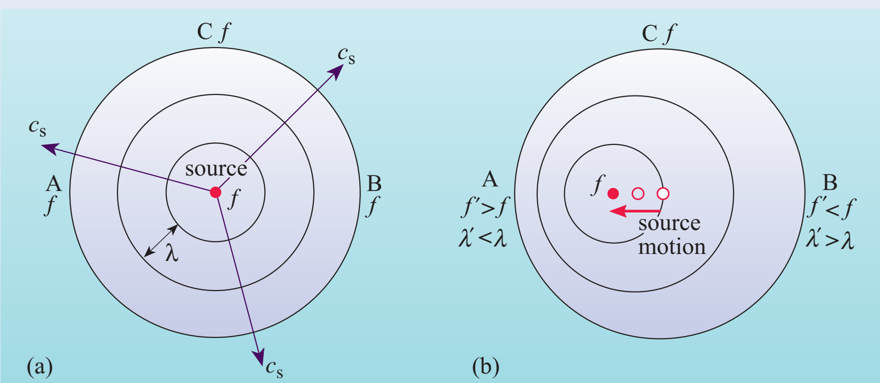

Figure 3.5 The Doppler effect with a car horn. (a) The car is stationary. (b) The car is in motion: an observer at A hears the horn at a higher pitch than the car driver; an observer at B hears it at a lower pitch than the driver; and an observer at C hears it at the same pitch as the driver.

sign of the difference we can tell whether the star is moving towards or away from us; we can thus obtain the radial velocity. (For light, or any other form of electromagnetic radiation, Equation 3.3 is an approximation requiring $v \ll c$, a condition met by stellar radial speeds.)

In the case of stars, it is more usual to work in wavelengths rather than frequencies. Using $\lambda = c/f$, we can express Equation 3.3 as

$$v_r = c \times (\lambda' - \lambda)/\lambda \qquad (3.4)$$

Where λ' is the observed wavelength.

- If the observed wavelength is longer than the emitted wavelength, in which direction is the star moving?
- It is moving away from us.

Increases in wavelength are called **red-shifts** from the days when observations at visible wavelengths dominated astronomy; at visible wavelengths an increase in wavelength takes us towards the red end of the spectrum. Likewise, decreases in wavelength are called **blue-shifts**. These shifts are collectively called **Doppler shifts**.

Radial velocities are roughly of the same order as transverse velocities. The two velocities together specify the overall motion of the star through space with respect to us. This **space velocity** is given (using Pythagoras's theorem, see Figure 3.3b) by

$$v = \sqrt{(v_t^2 + v_r^2)} \tag{3.5}$$

These overall motions are not entirely random, but are partly related to the large-scale motions in our Galaxy and by the grouping of many stars in clusters, which you will meet later in Section 3.2.4. Indeed, whether a star belongs to a cluster can often be decided by comparing its motion through space with that of the cluster members.

3.2.2 How far away are the stars?

Throughout history, people have been fascinated by the question of how far away are the stars. What gulf of space separates the stars from the Sun? Are all the stars at the same distance? The stars are so remote that the measurements that provided definitive answers to these questions began to emerge only in the first half of the 19th century. In addition to helping us to establish cosmic architecture, we also need to know stellar distances in order to investigate the stars as individual bodies. For example, until their distances were known, no progress could be made on determining either the size of the stars, or how much energy they radiated. Thus, one star could appear to us to be much brighter than another simply because it is far closer.

So, how do we measure the distances to the stars? There is a range of methods, each appropriate for different distances, which build up the astronomical distance scale, from the closest stars to the most distant galaxies. You will come across a number of these later. Here we shall describe the technique which was first to yield a stellar distance, is still the most accurate for the nearer stars, and provides us with an important unit of distance widely used in astronomy.

Figure 3.6 illustrates the principle of the method. When the Earth is at point A in its orbit, the relatively nearby star appears as shown against the background of far more distant stars. Six months later, from point C, the position of the star against the distant background appears to have shifted. You can easily demonstrate this kind of shift to yourself by holding up a finger at arm's length in front of you, and viewing it against a distant background alternately with one eye and then the other.

Returning to the stars, from the apparent shift in position of the nearby star, the angle 2θ in Figure 3.6 is measured by an Earth-based observer, and the distance d to the star is then given by

$$d = b/(\theta/\text{radian})$$

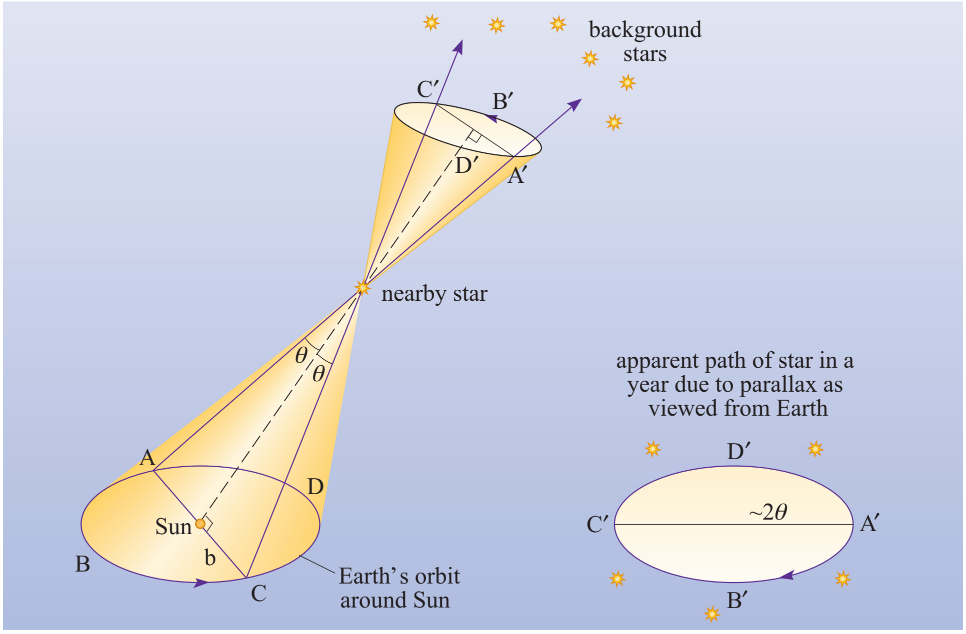

Figure 3.6 Trigonometric parallax. A nearby star's position changes relative to more distant stars as the position of the observer (on the Earth orbiting the Sun) changes. The star tracks out a path on the sky called the parallactic ellipse.

where b is the distance from the Earth to the Sun, and where we have again used the small-angle approximation (as in Equation 3.1), θ always being very small. Thus, if we know b then we can obtain d. The distance b can be obtained in a variety of ways, nowadays by measuring the times it takes radar pulses, which travel at the well-known speed of light, to return to the transmitter after being reflected off various bodies in the Solar System. The details will not concern us: the important point is that b is known, and therefore the distance d to the nearby star can be obtained. This method of obtaining stellar distances is called **trigonometric parallax**, this being the change in direction to an object as a result of a change in the position of the observer. In everyday parlance it is just called parallax.

The distance b, from the observer to the Sun is always very close to 1 astronomical unit (i.e. 1.50×10^{11} m), which is the *average* distance from the Earth to the Sun. If $b = 1$ AU we define the angle θ as equal to p, the **stellar parallax** (often abbreviated to **parallax**) and so

$$(d/\text{AU}) = 1/(p/\text{radian}) \qquad (3.6)$$

In the most precise measurements a small correction must be made to account for the fact that the Earth's orbit is not precisely circular.

The important unit of distance referred to earlier is the **parsec** (pc), defined as the distance d corresponding to a stellar parallax of 1 arcsec, as shown in Figure 3.7. Thus, by definition,

$$d/\text{pc} = \frac{1}{p/\text{arcsec}} \qquad (3.7)$$

With 206 265 arcsec in a radian, it is not too difficult to show, from Equations 3.6 and 3.7, that 1 pc = 206 265 AU (and hence 1 pc = 3.09×10^{13} km).

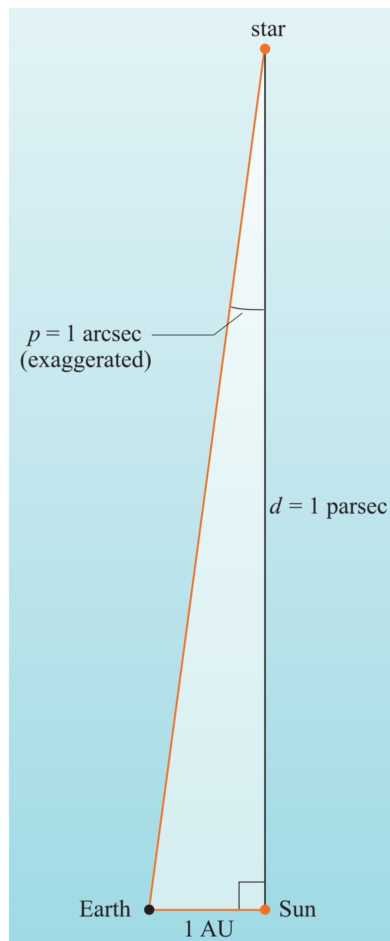

Figure 3.7 The definition of the parsec.

■ Is 'parsec' a reasonable name for this new unit?

❑ Given that it is made up from *par*allax and arc*sec*, it *is* a reasonable name.

During the course of a year the apparent position of the nearby star tracks out a tiny ellipse on the sky called the **parallactic ellipse** (see Figure 3.6).

■ What will be the apparent path of a star if it is: (a) in the same plane as the Earth's orbit, and (b) if it is in a direction perpendicular to the plane of the Earth's orbit?

❑ (a) A line of angular length 2θ, (b) a circle of angular radius θ.

Stellar parallaxes are all very small and were only discovered after many failed attempts. Some of these attempts led to the discovery of other important phenomena such as proper motion (Section 3.2.1) by Edmund Halley, and binary stars (Section 3.2.3) by William Herschel. James Bradley observed oscillations in the apparent position of the star γ Draconis over the period of a year with an amplitude of 20 arcsec. He had not, however, observed parallax since the maximum displacement occurred three months too late. He had discovered the **aberration** of starlight. The apparent direction of arrival of light from the star was the result of the combination of the speed of the Earth in its orbit and the finite speed of light from the star (analogous to running in the rain with an umbrella, the telescope had to point slightly in the direction of motion of the Earth to 'catch' the starlight – see Figure 3.8). All these effects have to be accounted for when attempting to measure parallax.

The first published measurement of parallax was by the German astronomer Friedrich Wilhelm Bessel in 1838. He found the parallax of the star 61 Cygni ('sig-nee') to be 0.314 ± 0.020 arcsec. (It is now known to be 0.287 arcsec.) This is a very small angle, 6000 times smaller than the angular diameter of the Moon. It is therefore not surprising that science had to wait so long for this first parallax measurement. It gave us the first measurement of a distance to a star (other than the Sun).

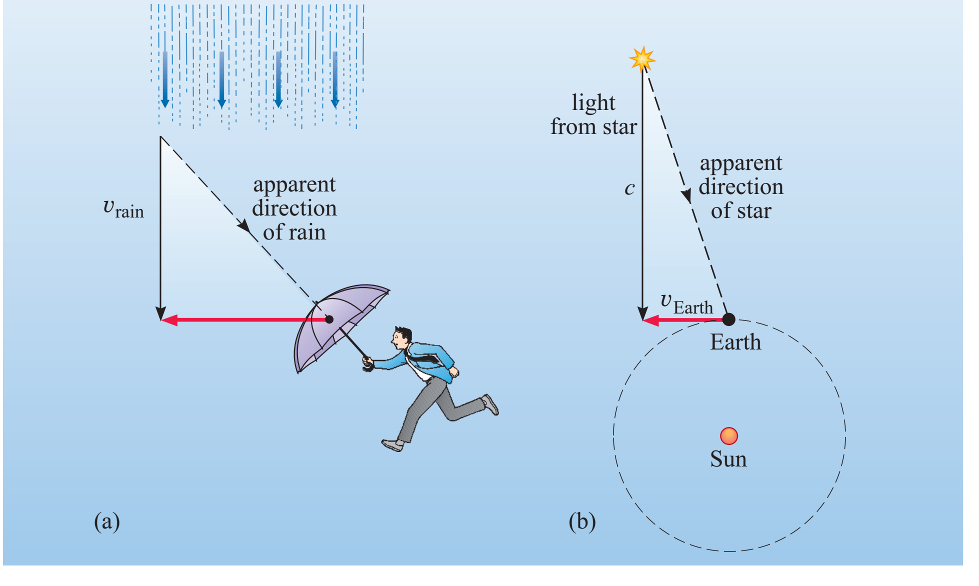

Figure 3.8 (a) A pedestrian running in the rain needs to point an umbrella forwards to keep dry. The apparent direction of the rain is the result of the sum of the raindrops' velocity and the pedestrian's velocity. (b) Similarly, an observer on the Earth needs to point a telescope slightly in the direction of the Earth's velocity in its orbit to see a star. Since the velocity of light, c, is much larger than the Earth's orbital velocity the shift in position is very small. This effect is called aberration.

JAMES BRADLEY (1693–1762)

James Bradley (Figure 3.9) studied theology at Oxford and became a clergyman before resigning to become Savilian Professor of Astronomy at Oxford and later he succeeded Edmund Halley as Astronomer Royal. Although he did not measure the motion of stars due to parallax, his discovery of the aberration of starlight confirmed the Copernican concept of a moving Earth. He was reluctant to publish his work until he had fully confirmed his results – his careful observations, which resulted in the discovery of the wobble in the Earth's rotation axis due to gravitational interactions with the Moon, lasted almost 20 years!

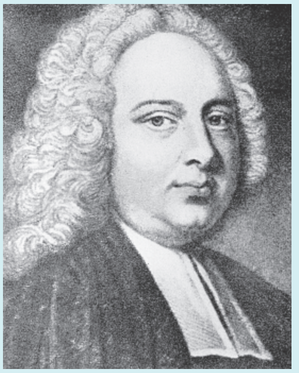

Figure 3.9 James Bradley. (Science Photo Library)

QUESTION 3.1

Calculate the distance to 61 Cygni, using the current value for its parallax, expressing your answer in parsecs, AU, and metres. How much further away from us is 61 Cygni than the Sun?

The largest parallax is for the star Proxima Centauri, which is therefore the nearest star, though with a parallax of only 0.772 arcsec it is 1.3 pc away, which is nearly 270 000 times further from us than the Sun. Clearly, a large gulf of space separates us from the stars. It's not that we are particularly isolated: the distances between the stars, at least in our part of the Milky Way, are generally of the same order as the distance to Proxima Centauri.

FRIEDRICH WILHELM BESSEL (1784–1846)

Friedrich Bessel (Figure 3.10) began work in Germany as an apprentice in an exporting company but dreamed of escape though travel. His study of languages, geography and navigation led to mathematics and eventually astronomy. Heinrich Olbers (1758–1840, famous at that time for his work on the discovery of asteroids) obtained a post for him at Lilienthal Observatory after receiving a paper written in 1804 on the motion of Halley's comet. This was followed by a commission from the Prussian government to build the first large German observatory at Königsberg (now Kaliningrad, Russia). After its completion in 1813 he remained its director until his death and made major advances in the accuracy of positional astronomy. In addition to the discovery of parallax, resulting from the determination of precise positions of 50 000 stars, he developed Bessel functions (a mathematical tool which he used for studying planetary motions) and suggested the presence of Neptune from perturbations in Uranus' orbit. Olbers claimed his own greatest contribution to astronomy was to lead Bessel to become a professional astronomer.

Figure 3.10 Friedrich Bessel. (Science Photo Library)

Table 3.1 lists the ten nearest stars after the Sun. Parallaxes smaller than 0.01 arcsec are very difficult to measure from Earth-based observatories, so we can obtain distances with useful accuracy (within a range of a few tens of parsecs) for only a few hundred stars. We can do far better from space, above the troublesome effects of the Earth's atmosphere.

The Hipparcos satellite was named to commemorate the Greek Astronomer Hipparchus (*c.*170–120 BC) who produced the first accurate star catalogue.

The Hipparcos satellite (*Hi*gh *P*recision *Par*allax *Co*llecting *S*atellite) launched by the European Space Agency (ESA) in 1989, measured the parallaxes of 118 000 stars to an average precision of better than 0.001 arcsec (1 *milli*-arc second). In addition, proper motions were measured, with a precision of 0.001 arcsec yr^{-1}, permitting derivation of accurate three-dimensional positions and motions of stars out to beyond 100 pc. Hipparcos also measured the positions, motions, brightnesses and colours of a further 2.5 million stars with lower precision.

Appendix A3 contains information on the 100 closest stars to the Sun.

Table 3.1 The ten nearest stars after the Sun.

Name	Parallax /arcsec	Distance /pc	Distance /ly	Proper motion /arcsec yr^{-1}	Comment[a]
Proxima Centauri	0.772	1.30	4.22	3.9	triple system
α Centauri A	0.747	1.34	4.36	3.7	
α Centauri B	0.747	1.34	4.36	3.7	
Barnard's Star	0.547	1.83	5.95	10.4	
Wolf 359	0.419	2.39	7.77	4.7	
Lalande 21185	0.393	2.54	8.28	4.8	
Sirius A	0.380	2.63	8.57	1.3	binary system
Sirius B	0.380	2.63	8.57	1.3	
L-726–8A	0.373	2.68	8.73	3.4	binary system
L-726–8B	0.373	2.68	8.73	3.4	

[a] In a binary system, two stars are in orbit around each other. In a triple system there are three such stars (see Section 3.2.3).

The Gaia satellite, planned for launch by ESA in about 2012, will revolutionize our view of our Galaxy, measuring the positions and motions of the billion brightest objects in the sky. Its design goal is to determine parallaxes to an accuracy of 1 *micro*-arc second (0.000 001 arcsec) (giving distances of stars out to tens of thousands of parsecs), and proper motions to an accuracy of 1 micro-arcsec yr^{-1}. While most of the objects detected by Gaia will be stars, allowing us to map the three-dimensional structure and motion of the whole of our Galaxy, it is also expected to find, from their motions, over 150 000 new asteroids in our Solar System and 50 000 stars in our Galaxy possessing planets.

Finally, note that the parsec is not used solely to express the distances to the stars: it is commonly used to express *any* distance greater than the size of the Solar System. Another unit commonly used for such distances is the **light-year** (ly), which is the distance that electromagnetic radiation would travel in a vacuum in a year, and is equal to 0.307 pc. Neither the light-year nor the parsec is an SI unit but they result in more reasonable numbers for stellar distances; the parsec is the preferred unit for professional astronomers.

3.2.3 Binary and multiple star systems

William Herschel observed stars which were apparently very close to each other in the sky in an attempt to identify tiny relative motions that might be due to stellar parallax. This method would only work if the stars were chance 'line of sight' alignments with one nearby and one distant. He failed to measure parallax, but catalogued around 700 pairs of stars within 2 arcsec of each other, far more than would be expected by chance, and showed that several of them were orbiting each other. In fact, John Michell (1724–1793) had earlier calculated that the probability of chance alignments of close double stars was extremely small.

THE HERSCHEL FAMILY

Sir (Frederick) William Herschel (1738–1822) (Figure 3.11a) was born in Germany but emigrated to England after visiting with the regimental band of the Hanoverian guards. His musical activities funded his interest in astronomy as he constructed ever larger telescopes. He made a systematic survey of the sky, discovering the planet Uranus in 1781, which he originally named Georgium Sidum in honour of King George III, which no doubt helped him to be appointed court astronomer. In addition to his studies of double stars, he also discovered two satellites of Uranus and Saturn, accurately determined the rotation period of Mars and deduced the Sun's motion through space, relegating the Solar System from its favoured place at the centre of the Universe. He used prisms and thermometers to detect radiation in the solar spectrum but beyond the red end of the visible spectrum and hence discovered infrared radiation.

William's sister, Caroline Herschel (1750–1848) (Figure 3.11b), as a girl in Germany, was denied an education and was destined to become a housekeeper. In 1772 she moved to England to become housekeeper for William and also became his observing assistant, compiling catalogues of 2500 nebulae and 1000 double stars. By quizzing William over the breakfast table she learnt spherical trigonometry and logarithms and when he was away she searched the sky for comets, discovering eight. Like her brother, she received several prestigious awards – the first when she was 78 and the last on her 96th birthday!

William's only son, Sir John Frederick Herschel (1792–1871) (Figure 3.11c), continued the astronomical dynasty. From 1834 to 1838 he systematically mapped the Southern skies from Cape Town in South Africa. He invented an instrument which allowed him to adjust the brightness of an image of the full moon to match a star under observation – a great advance in stellar photometry. His General Catalogue of Nebulae and Clusters is still, in updated form, the standard reference (and known as the NGC). John Herschel's fame extended beyond the scientific community and he was recognized as a great public figure of his time.


(a)


(b)


(c)

Figure 3.11 (a) William Herschel, (b) Caroline Herschel, (c) John Herschel. (Royal Astronomical Society)

You may have noticed from Table 3.1 that seven of the nearest ten stars are in binary or triple star systems. In fact over half the known 'stars' are binary systems, and so such systems account for over two-thirds of individual stars. (Note that the word 'star' applies to an apparently single point of light in the sky, as well as to truly individual stars. A binary system is often called a binary star, and so a **binary star** actually contains two stars!)

■ Why do most stars appear to be single when observed with the unaided eye or a telescope?

❑ They are so far away that even if the two stars in a binary are separated by large distances their *angular* separations are so small they cannot be distinguished (we say that the binary star cannot be resolved).

When both stars in a binary system can be seen as distinct points of light, they are called **visual binary systems**, though nowadays the observations are made by electronic or photographic imaging, and not by the eye at the telescope eyepiece. One of the most famous visual binaries is Sirius, the brightest star in the night sky as seen from the Earth. (When we refer to a binary star by its name, e.g. Sirius, we have not distinguished between the two components, Sirius A and Sirius B. Often, one star (A) is much brighter than the other so the luminosity of the combined binary star is essentially the same as its brightest component.) With a large telescope we can see two stars, the brightest (primary) star Sirius A, and the much fainter Sirius B, first seen by the US telescope maker Alvan G. Clark (1832–1897) in 1862, while he was testing a new telescope lens. The orbital period of each with respect to the other is 50 years, and their angular separation on the plane of the sky has a maximum of 11.5 arcsec. Examples of binary stars visible in a small telescope are: γ And, Albireo (β Cyg) and Mizar (ζ UMa). (Mizar (ζ UMa) is also a pair with Alcor which can be distinguished with the unaided eye.) Figure 3.12 shows the orbit of one star in a binary with respect to the other (the small irregularities arise from uncertainties in the observations – the orbit is actually very smooth). In reality, both stars are moving about their common centre of mass that

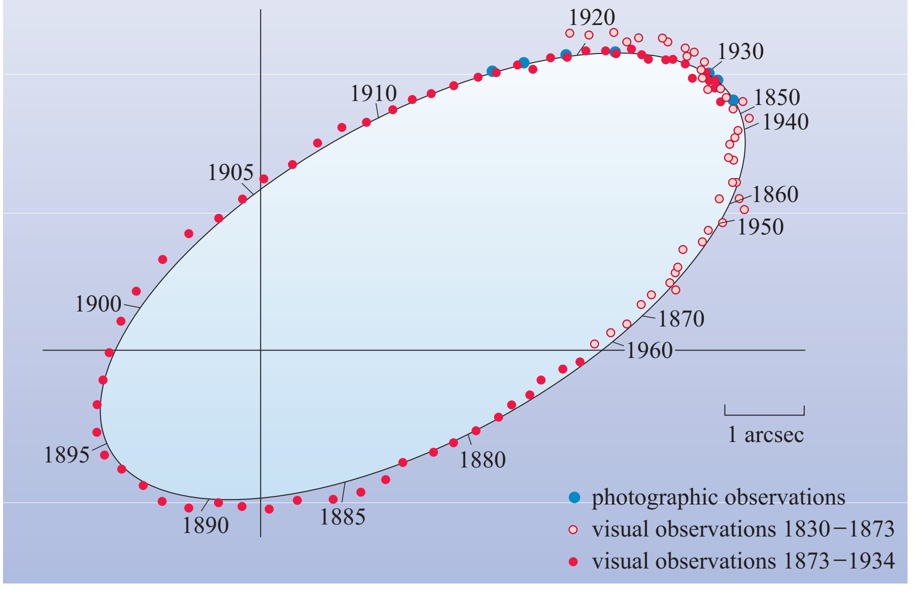

Figure 3.12 The orbit in the sky of one star relative to another in the visual binary system 70 Oph. The orbital period is 88 years. (Strand, 1973)

is moving with its own space velocity. Visual binaries are of immense value in determining the masses of individual stars (Section 3.3.7) but can only be identified if they are close to the Sun and if the stars are well separated.

Other binaries reveal their nature indirectly.

Spectroscopic binaries reveal themselves from the changing Doppler shifts (Section 3.2.1) in the spectral lines of one or both stars as they orbit each other (Figure 3.13). The majority of known binary stars have been identified by this method since there is no requirement for them to be close to the Sun, only that they are bright enough for good quality spectra to be obtained.

- Is there any other requirement of the binary star system for it to be identified by relative Doppler shifts of the spectral lines?

- The orbital plane of the two stars must not be at right angles to the line of sight. If this is the case then there will be no component of motion of the stars along the line of sight and therefore no radial velocity component and hence no relative Doppler shift.

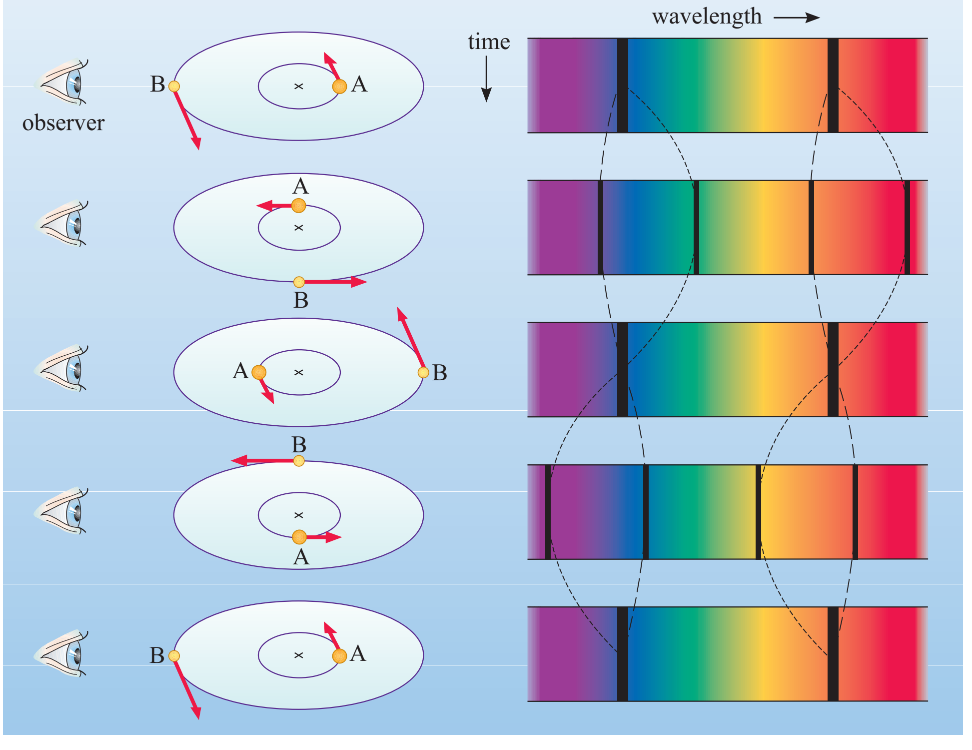

Figure 3.13 Identification of a spectroscopic binary star from Doppler shifts of spectral lines as the stars orbit their common centre of mass (marked with a cross). The diagrams on the left show the geometry of the stars in their orbits. On the right is a schematic representation of part of the spectrum of the stars (the spectra cannot be separated as the stars are too close together to be resolved).

There are also **eclipsing binaries**, in which we view the orbit so close to edge-on that one star is seen to pass in front of the other, and the observed brightness of the 'star' will dip (as you will see in Figure 3.41). These are rare but a famous example that can be seen with the unaided eye is Algol (β Per), which exhibits a rough halving of its luminosity for a few hours every 69 hours. In some mythologies, Algol is the winking eye of a demon.

3.2.4 Star clusters

A brief glance at the night sky will confirm that the stars are not distributed uniformly across the sky. If you are able to observe from a dark site (unfortunately not so simple these days due to the light pollution from street lighting) you will see that the greatest concentration of stars coincides with a faintly glowing band which circles the sky – the Milky Way. Binoculars or a telescope reveal that this is the light of myriads of individual stars. It is immediately apparent that the Galaxy is flattened, but less clear that our Sun is located well away from the centre (see Figure 0.1 in the Introduction). This is due to the presence of obscuring dust towards the centre of the Galaxy. Counting stars provides a tool for probing the size and thickness of absorbing regions and hence for mapping of the distribution of dust in the Galaxy. (The effect of interstellar gas and dust on starlight is discussed in Sections 4.3.2 and 4.3.3.)

Intermediate between the large-scale distribution of stars on the sky and the closely bound binary or multiple star systems, are groups of hundred or thousands of stars. Any group with more than a few members is called a **star cluster**. The Pleiades ('ply-a-dees'), in the constellation of Taurus visible on winter nights in the Northern Hemisphere, appears as a group of five or more stars to the unaided eye. Binoculars reveal many more; it contains around 3000 stars (see Figure 3.14).

Figure 3.14 The open cluster, the Pleiades, is easily visible to the unaided eye and is about 80 million years old. The blue light is starlight scattered by dust close to the stars (see Section 4.3.3). (ROE/AAO/ D. Malin)

The Pleiades is an example of an **open cluster**. They typically contain a few hundred stars, are sometimes irregular in shape, often contain very hot luminous stars, generally lie close to the plane of the Galaxy and are often associated with dust and gas clouds. These clusters contain barely enough mass to be bound by gravity and will eventually dissipate. As you will see later (Section 4.2.5), open clusters play a vital role in understanding the process of stellar evolution. Since all the stars were formed at more or less the same time in the same region of space they provide a 'snapshot' of a collection of stars which are all the same age, formed from the same material. They also provide a way of comparing the relative properties of stars since all the stars are effectively at the same distance from us.

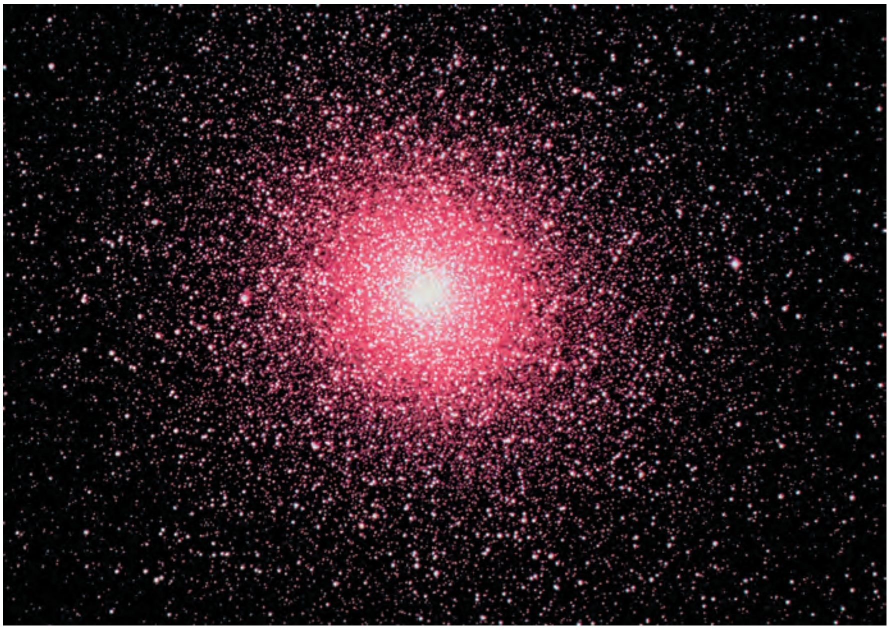

Figure 3.15 The globular cluster 47 Tucanae. This cluster is visible to the unaided eye from the Southern Hemisphere as a faint smudge but requires a telescope to see individual stars. (NASA/ESA)

A different type of star cluster, with very different properties, is observable in our Galaxy. **Globular clusters** like 47 Tucanae (Figure 3.15) contain many thousands of stars, tightly bound by gravity into a spherical shape. They are distributed spherically about the Galaxy, have little or no gas and dust, and usually contain no very hot stars. The properties of globular clusters imply great age and they provide a tool for probing the structure and history of material in our Galaxy. They are also sufficiently bright to be identifiable in other galaxies, providing a method of determining the distances to those galaxies (assuming their properties are the same in external galaxies).

3.3 The stars as bodies

3.3.1 How big are the stars?

The Sun is a spherical body with a radius about 110 times that of the Earth. The volume ratio is a more impressive million or so. What of the other stars? Is there a great range of sizes? Is the Sun typical, or exceptionally small, or particularly large? Of one thing we can be certain: with a few readily identified exceptions, the stars are spherical, just as we expect for massive bodies dominated by their own gravity. Thus we can characterize size by radius.

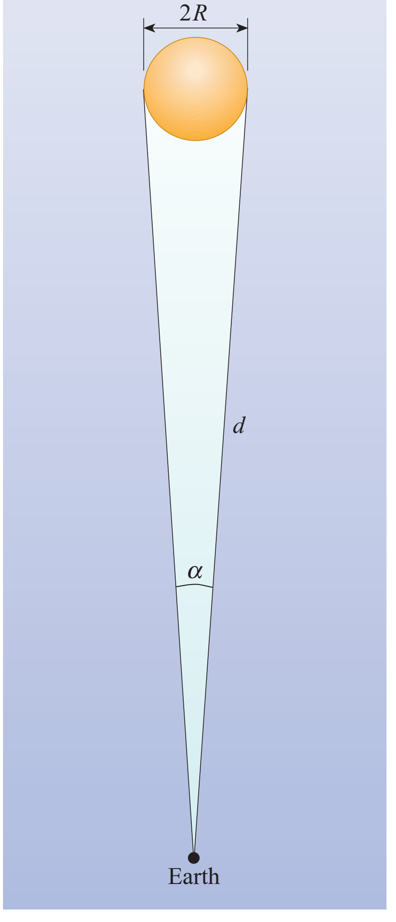

Figure 3.16 A star's angular diameter and radius.

For a star of known distance d, the most direct way to calculate the radius is to measure the angular diameter α. Then, as Figure 3.16 shows, the star's radius is given by

$$R = [(\alpha/2)/\text{radians}] \times d \tag{3.8}$$

provided that α is small, which it certainly always is!

If we have a telescope that can show a star as a disc then we can measure α. Alas, this is possible for only those stars with particularly large values of α. This is because a telescope, even when looking at an object of *negligible* angular size, will produce an image that is not a point but a blur of finite size. Thus, a star has to

have a sufficiently large angular diameter to produce an image with a size that considerably exceeds that of the point blur. This blur arises partly from turbulence in the Earth's atmosphere (called 'seeing') and partly from fundamental optical limits that are more severe the smaller the main mirror or lens in the telescope. Modern techniques such as adaptive optics can be used to overcome some of the effects of atmospheric turbulence by constantly adjusting the shape of the telescope's mirror. Alternatively, telescopes can be combined to make, in effect, a single large telescope using a technique called long baseline inteferometry. Using such techniques, the angular diameters of a few hundred stars have been measured, from the star with the largest angular diameter, namely Betelgeuse, prominent in Orion (Figure 3.1), with $\alpha = 0.050$ arcsec, down to as little as 0.0004 arcsec for ζ Puppis – equivalent to the width of a human hair at a distance of a few kilometres! (The diameter of Betelgeuse is so large that surface details have been imaged – see Figure 3.17.)

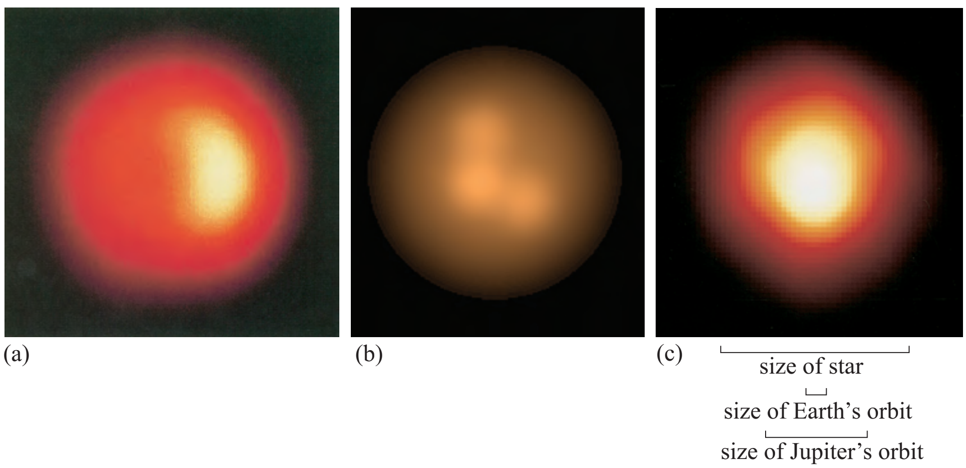

Figure 3.17 Betelgeuse is one of the few stars which is large enough and close enough for structure to be imaged on its surface. It is nearly a thousand times larger than the Sun and 20 times more massive, and is a red supergiant. It is 131 pc (430 light-years) away. Structure is visible on the surface of Betelgeuse when imaged using the Cambridge Optical Aperture Synthesis Telescope consisting of five small telescopes (b). In fact Betelgeuse is so large that it can be imaged using parts of a single telescope as an interferometer (as in (a), taken using the 4.2 m William Herschel Telescope) or, at lower resolution, using the Hubble Space Telescope (c). The bright regions, probably due to convection, have changed between the observations. ((a) P. Warner/MRAO/William Herschel Telescope; (b) courtesy COAST Group; (c) NASA)

These measured values of α, multiplied by d as in Equation 3.8, have yielded stellar radii ranging from rather less than that of the Sun, to about 1000 times greater. Thus there is a great range of stellar radii, and the Sun is a rather small star.

A few hundred stars are rather a small sample. However, there are other ways of obtaining stellar radii, and we shall meet one of these later in this chapter.

QUESTION 3.2

Betelgeuse is 131 pc away. Calculate its radius, in metres and in solar radii.

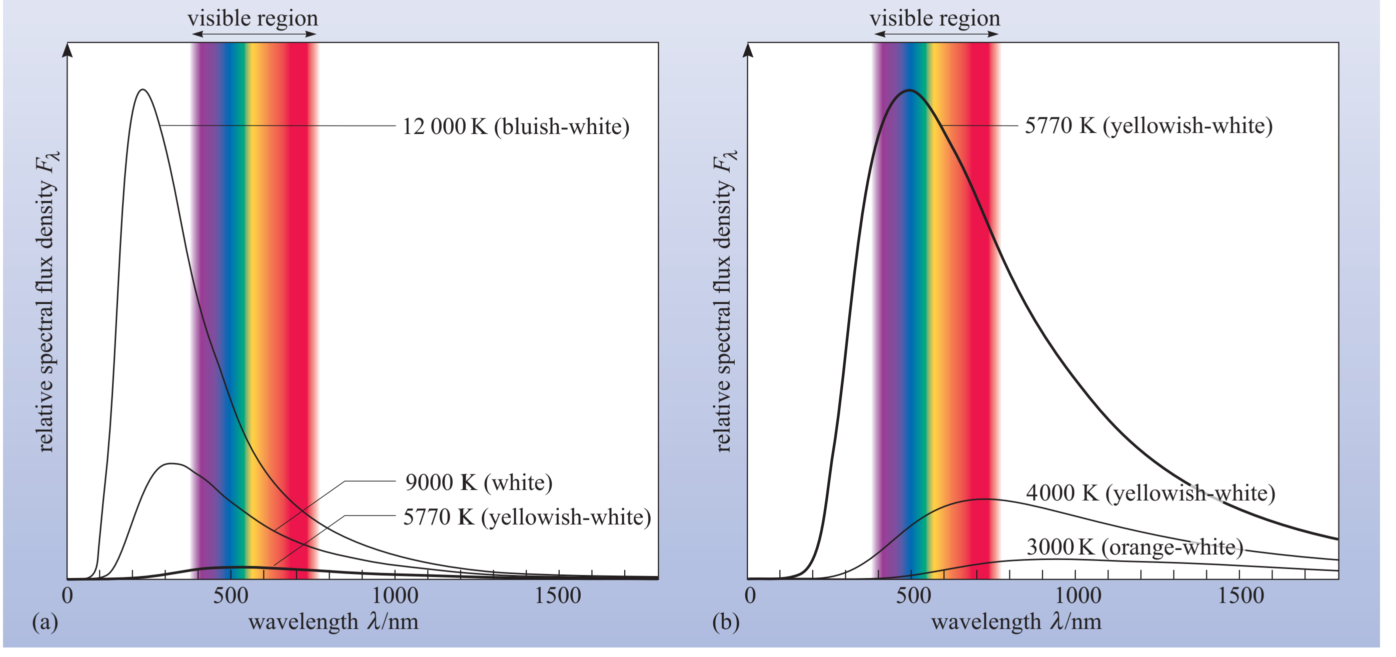

Figure 3.18 Black-body spectra at different temperatures. Each source is of the same size, and at the same distance from the detector that measures the flux density. Note that the vertical scale for the set of spectra in (a) is greatly elongated compared with that for the set in (b).

3.3.2 How hot are the stars?

You have seen in Chapter 1 that the radiation that we observe from the Sun originates largely from the photosphere, and that its distribution over wavelength – its spectrum – is close to that of a black body. If the distribution were *exactly* that of a black body then there would be a single temperature for the emitting region. Figure 3.18 shows some black-body spectra at different temperatures. The Sun's spectrum is not exactly that of a black body, and so we cannot give the photosphere a unique temperature. However, the spectrum at 5770 K in Figure 3.18 is a good fit to the solar spectrum, which means that the temperature of the source of the photospheric radiation is nowhere enormously different from 5770 K. Thus the radiation from the Sun comes largely from a region with a temperature around 5770 K. This is the Sun's 'surface' temperature.

The other stars, too, have spectra that are not very different from black-body spectra. Therefore we can obtain meaningful photospheric temperatures from well-fitting black-body spectra. A crude way of doing this is from the star colour that we perceive: from Figure 3.18 we see that white and bluish-white stars are hotter than the yellowish-white Sun, and orange-white stars are cooler than the Sun. A better way is to compare the radiation that the star emits at two different wavelengths, as you saw for the Sun in Question 1.10, or over two different wavelength *ranges*, as illustrated in Figure 3.19. From such a comparison, we can obtain the temperature of the photosphere. This is the basis of the **photometric method** of temperature determination. Note that we do not need to measure the complete spectrum: one ratio suffices.

However, for many individual stars, more accurate values of photospheric temperatures are obtained by the **spectrometric method** which is based on examination of the spectral absorption lines in starlight. The lines of interest are those formed, as in the Sun, by absorption in the star's upper photosphere and

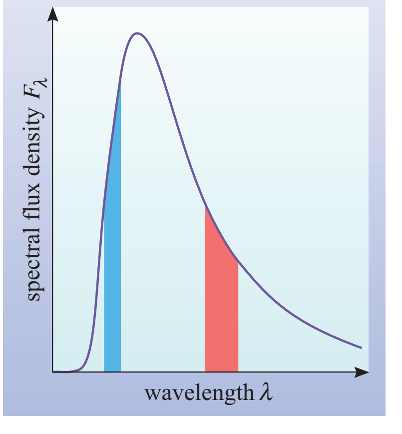

Figure 3.19 The photometric method of obtaining photospheric temperatures. The ratio of the amount of energy measured in two different wavelength regions (shaded) is uniquely defined by the temperature if the object emits like a black body.

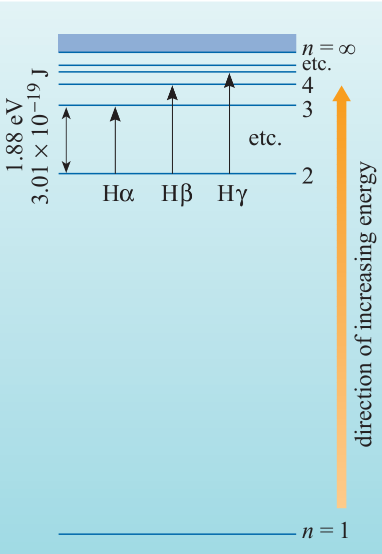

Figure 3.20 Electron transitions for the hydrogen Balmer absorption lines.

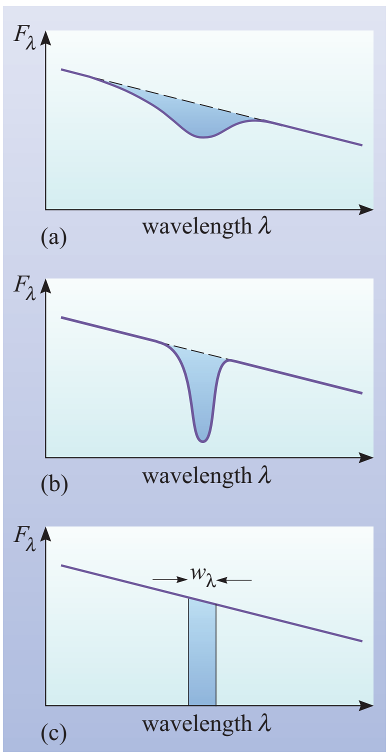

Figure 3.21 The definition of equivalent width w_λ. The shaded area between the vertical lines in (c) is the same as that bounded by each of the two spectral lines in (a) and (b). Therefore all three lines have the same equivalent width w_λ.

in the region just above it. For example, consider the hydrogen **Balmer absorption lines**. This is the name of the group of lines that you met in Chapter 1, with the individual labels Hα, Hβ, Hγ, and so on. They correspond to the electron transitions in the hydrogen atom as shown in Figure 3.20. For all the Balmer lines the lower energy level is that for which $n = 2$. If the temperature is very low, then nearly all the electrons are in the $n = 1$ level, and the Balmer lines are very weak. If, on the other hand, the temperature is very high, then nearly all the hydrogen atoms are ionized, and the lines are again weak. Between these two extremes, there will be a range of temperatures at which a larger proportion of the electrons are in the level $n = 2$ and transitions from this level will be common and hence the Balmer lines will be strong.

The strength of a spectral line is defined by the amount of radiation (as measured by the flux density) removed from the spectrum by the absorbing material (or added in the case of an emission line). Spectral lines may be broad and shallow or deep and narrow but have the same strength (in fact the shape of the line tells us a lot about the conditions in the gas which caused it as you will see later). The **equivalent width** of a line provides a quantitative measure of the strength of a spectral line. It is the width of a section of nearby continuum which has the same area as that between the spectral line and the continuum, shown in Figure 3.21. The variation of line strength with temperature is shown schematically in Figure 3.22.

Clearly there is a basis here for measuring temperature. However, there are two difficulties. Figure 3.22 indicates one of them.

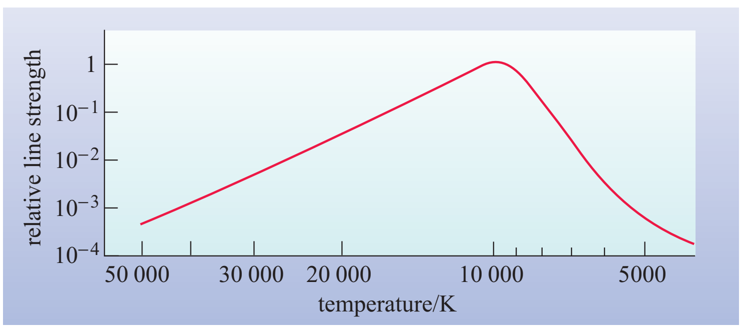

Figure 3.22 The strength of the hydrogen Balmer absorption lines versus photospheric temperature. Note the temperature scale is logarithmic (this figure is often plotted with a scale like that of Figure 3.23).

- What is this first difficulty?
- A given line strength corresponds to two temperatures.

Thus, a very hot and a very cool star will have Balmer lines of similar strength. We overcome this difficulty by observing other absorption lines that, like the hydrogen Balmer lines, are also particularly sensitive to temperature, but which have a different variation of strength with temperature from that of the hydrogen Balmer lines. Figure 3.23 shows schematically a collection of such line strengths.

- On the basis of Figure 3.23, what is the temperature of a star for which the hydrogen Balmer and (unspecified) helium lines have equal strength?
- About 20 000 K.

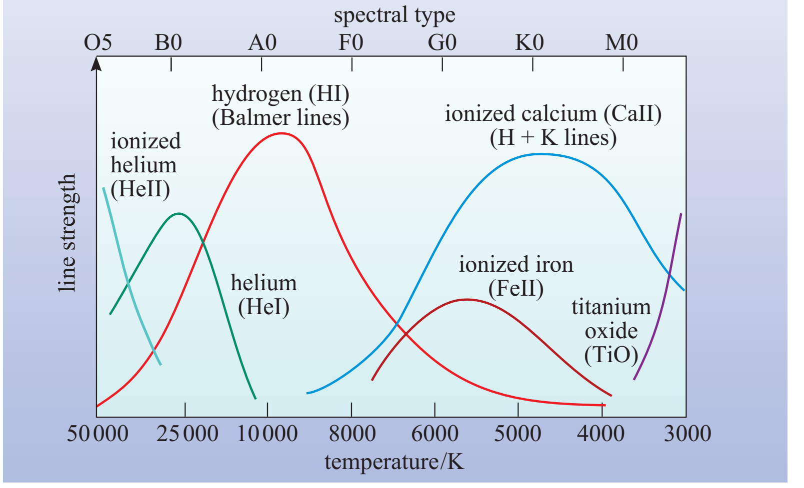

Figure 3.23 The strengths of various absorption lines versus photospheric temperature (note the temperature scale is not linear or logarithmic but in roughly equal steps of spectral type, introduced below).

The second difficulty is that the strengths of the various lines are also sensitive to the elemental abundances. Clearly, you would not see calcium lines at *any* temperature if there is no calcium present. We overcome this difficulty from spectral studies that reveal the composition of the region from which the absorption lines originate (Section 3.3.6). The elements chosen for temperature measurements are those that do not exhibit large variations in abundance from star to star.

With these and other difficulties overcome, the spectrometric method of obtaining photospheric temperatures was well established by the 1920s, mainly through the efforts of the US astronomer Annie Jump Cannon at Harvard University. On the basis of the strengths of their spectral lines, stellar spectra were classified by letter in a scheme called the **Harvard Spectral Classification**. Originally, spectra were sorted into groups labelled from A (strong hydrogen lines) to O and beyond (hydrogen lines weak or missing). Later, as work progressed, the underlying physical principles represented by Figures 3.22 and 3.23 exposed the inadequacy of this progression and the scheme was modified. Some of the groups were dropped, some were merged with others, and the remainder were ordered according to temperature, to give the scheme widely used today. In order of descending temperature the spectral classes are labelled O B A F G K M, which you should remember: one useful mnemonic is '*O*h *B*other, *A*n *F* *G*rade *K*ills *M*e!' (there are many alternatives!). The classes B to M are subdivided into spectral types 0, 1 … 9, with 0 at the hotter end. For example, F9 and G0 are spectral types of stars that are not very different in temperature. Class O is (perversely) subdivided 5, 6, 7, 8, 9, 9.5(!), with 5 at the hotter end. Table 3.2 gives the temperatures around the beginning and the middle of each class. There is one further complication, the temperature for a particular spectral type also depends somewhat on luminosity (as you will discover in Section 3.3.4).

ANNIE JUMP CANNON (1863–1941)

Annie Jump Cannon (Figure 3.24) was one of the first women from the state of Delaware to attend university; she graduated in Physics from Wellesley College in 1884. She then spent a decade at home with her parents, unfortunately becoming almost totally deaf following scarlet fever. In 1894, after her mother's death she returned to Wellesley and in 1896 joined Harvard College Observatory. She was one of a group of women, paid 50 cents per hour, employed to classify stars and carry out calculations – they were known as 'computers'. She is responsible for the O, B, A, F, G, K, M, spectral classification scheme and during her lifetime classified some 400 000 stars this way. Her work was published in the nine volume Henry Draper Catalogue, 1918–1924. In 1925 she became the first woman to be given an honorary degree by Oxford University. Although recognized worldwide, she was not given an official position at Harvard until age 75.

Figure 3.24 Annie Jump Cannon. (Harvard College Observatory/ Science Photo Library)

Since the Harvard Classification Scheme was devised, a new spectral class of star has been defined, class L. These are cooler than spectral class M. You will meet these again in Section 6.4.2.

- On the basis of the strengths of its spectral lines, the Sun is spectral type G2. Is the corresponding temperature consistent with the value given earlier?
- From Table 3.2, G2 corresponds to a temperature of just under 6000 K, which is consistent with the value of 5770 K given earlier.

Stars are found across the full range of spectral types in Table 3.2, and so the Sun is not a particularly hot star. Far hotter is the bluish-white star Rigel A, which has a spectral type B8. (Rigel B and C are faint companion stars.) Somewhat cooler than the Sun is Betelgeuse (Figure 3.17), which has a spectral type M2, and looks orange-white. The colours of Rigel and Betelgeuse are discernible to the unaided eye and are apparent in Figure 3.1.

Table 3.2 Stellar spectral types, photosphere temperatures and prominent lines.

Spectral type	Temperature[a] /K	Most prominent lines (see Figure 3.23)
O5	40 000	ionized helium and other ionized atoms
B0 B5	28 000 15 500	neutral helium, hydrogen
A0 A5	9900 8500	hydrogen, some ionized metals
F0 F5	7400 6600	hydrogen, ionized calcium, iron and other metals
G0 G5	6000 5500	ionized and neutral calcium, iron, and other metals, hydrogen
K0 K5	4900 4100	neutral iron, calcium and other metals
M0 M5 M8	3500 2800 2400	titanium oxide, neutral calcium

[a] The exact temperatures of each spectral type also depend on luminosity to a lesser extent (see Section 3.3.4) so a range of values are possible for a given spectral type.

- Make a rough estimate of the temperatures of Rigel A and Betelgeuse.
- The approximate temperatures are 12 000 K for Rigel A and 3200 K for Betelgeuse.

Figures 3.25 and 3.26 show stellar absorption spectra belonging to various spectral types, in the form of graphs of relative spectral flux density (Section 1.3.2) versus wavelength: these are the sort of quantitative spectra now used to establish spectral type, and hence temperature. If we can establish a star's spectral type, then we can determine its average photospheric temperature with an uncertainty of only about 5%.

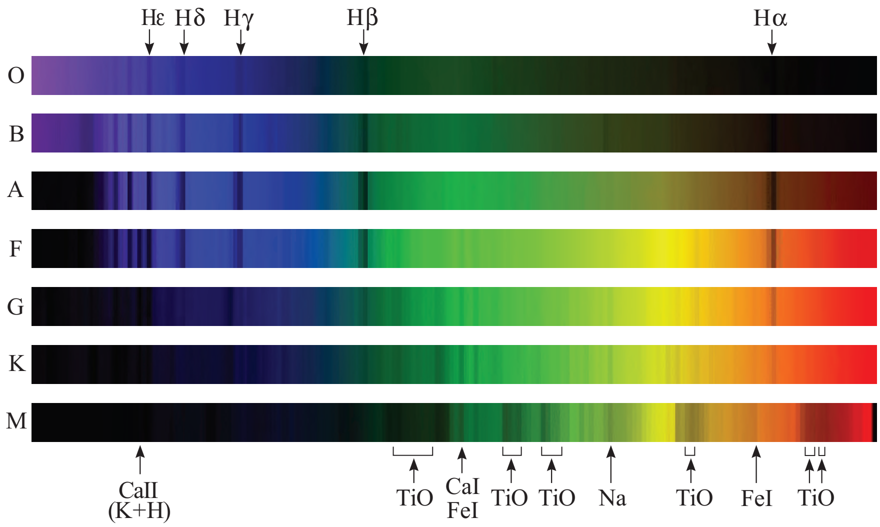

Figure 3.25 Stellar absorption spectra for different spectral types showing prominent absorption lines. (Kaufmann and Freedman, 1998)

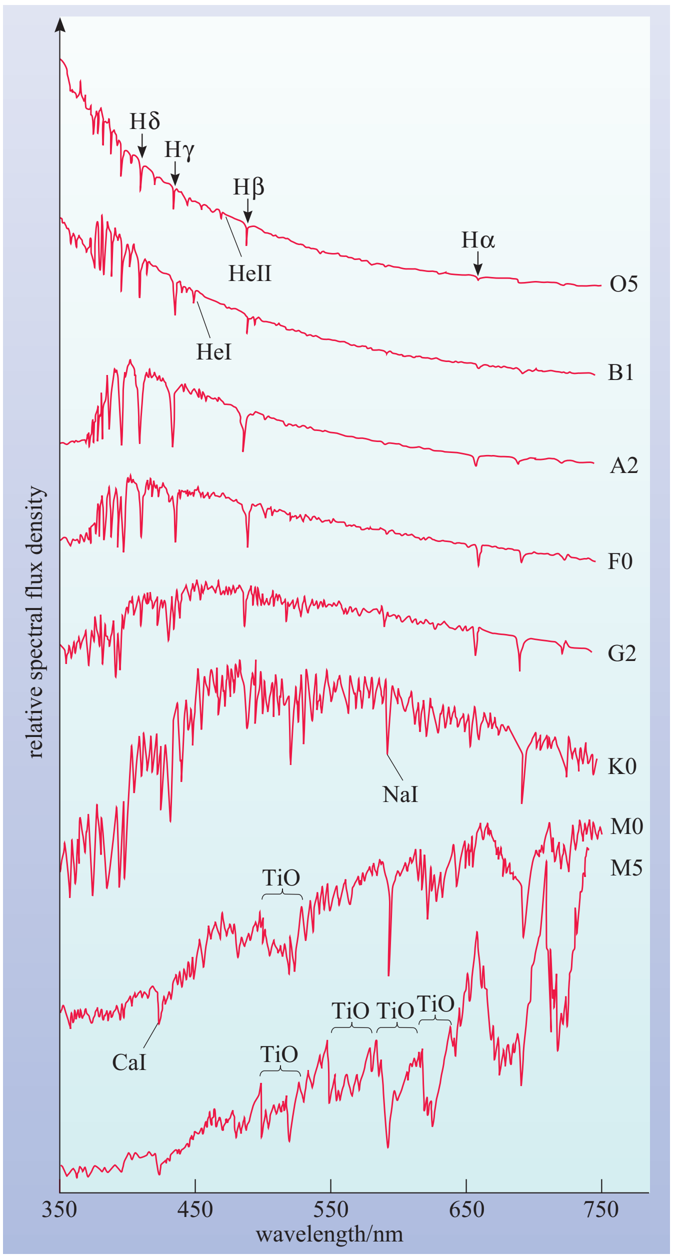

Figure 3.26 The stellar absorption spectra given in Figure 3.25 are more usually presented as graphs of relative flux density versus wavelength for ease of identification of the prominent absorption lines. The spectra have been plotted without spectral flux density scales and displaced vertically for clarity. (Kaufmann and Freedman, 1998)

QUESTION 3.3

From Figure 3.23 and Table 3.2, which spectral classes have weak hydrogen Balmer lines, and what are the corresponding temperatures? Compare your answer with the spectra in Figures 3.25 and 3.26.

3.3.3 How bright are the stars?

When we observe stars to determine their brightness we actually measure the amount of light from the star which is reaching us – its *apparent* brightness. If we want to study and compare the properties of the stars we really want to know their *intrinsic* brightness.

By intrinsic stellar brightness we mean the total amount of power a star radiates into space, over all wavelengths. This is called the **luminosity**, L, which in SI units is measured in watts. In Chapter 1 you saw that the Sun's luminosity is 3.84×10^{26} W, enormous by terrestrial standards, but how does it compare with the other stars?

The luminosity of a star depends on two of its properties that you have already met.

■ What do you think these properties are?

❑ The two properties are radius and temperature.

On the basis of everyday experience this is reasonable: as a ball of steel goes from red-hot to yellow-hot to white-hot it glows more brightly, and a pinhead of hot steel would radiate less power than a cannon ball of steel at the same temperature. We can readily develop a quantitative relationship between luminosity, radius, and temperature, because, just like the Sun (Chapter 1), any star radiates rather like a black body.

The power l radiated by unit area of a black body at an absolute temperature T (such as the kelvin scale) can be shown to be given by the simple equation $l = \sigma T^4$, where σ is called the **Stefan–Boltzmann constant**: in SI units, σ has the value $5.67 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$. For a star, we have the approximation $l \approx \sigma T^4$, where T is the average photospheric temperature. We can take a star to be spherical, and therefore its surface area is $4\pi R^2$, where R is the radius of the star. Thus the power emitted by the whole surface – the luminosity – is given by

$$L \approx 4\pi R^2 \sigma T^4 \qquad (3.9)$$

Therefore, if we know a star's radius and temperature then we can obtain its luminosity.

■ Can you see any problem with this approach?

❑ In Section 3.3.1 you saw that only a few hundred stars have had their radii measured. Therefore, this approach can yield the luminosities of only a few stars.

In reality, Equation 3.9 is often used to *determine* the radius when the luminosity is known from some independent method.

The luminosity of a star is an intrinsic property of the star, but how is the *apparent* brightness as seen by an observer affected by the distance of the observer? You will know that if you want to read a book at night, then the closer you hold the book to a source of light the greater the illumination. In more scientific terms, for a source of given luminosity, the closer the source to a surface facing it, the greater the flux density on the surface. **Flux density**, F, is the rate at which energy from a source crosses a unit area facing the source.

- How does it differ from spectral flux density, defined in Chapter 1?
- Spectral flux density is the rate at which energy from a source crosses unit area facing the source within a narrow wavelength range, divided by the width of the range.

The physical units of flux density are power per unit area, which in SI units is watts per square metre.

If we know the distance d from a star to the Earth then we can work out the luminosity by measuring the flux density. At a distance d from the star, its luminosity L is spread over a sphere of area $4\pi d^2$, as in Figure 3.27. For all practical purposes a star can be considered to radiate uniformly in all directions. Thus, at any point on the sphere the flux density is given by

$$F = L/(4\pi d^2) \quad (3.10)$$

We have assumed that the effects of interstellar matter between the star and the point where F is measured are negligible. Thus, if we measure F and we know d then we can obtain L, from a straightforward rearrangement of Equation 3.10, namely

$$L = (4\pi d^2)F \quad (3.11)$$

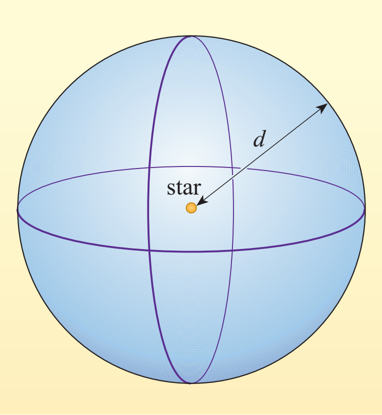

Figure 3.27 A spherical surface, radius d, centred on a star.

Conversely, by measuring the flux density received from a star of known luminosity L, we can obtain a value for the distance. From a further rearrangement of Equation 3.10

$$d = [L/(4\pi F)]^{1/2} \quad (3.12)$$

This method works for distances that extend well beyond the limit for trigonometric parallax; it merely (!) requires us to determine the luminosity by an independent method. There is a range of such methods, which usually involve determining the luminosity of a nearby object and making the assumption that all other objects with the same properties have the same luminosity. These so-called **standard candles** play a vital role in determining distances in astronomy and we will discover some of them later. The luminosity of stars can in fact be determined from their spectra alone if they are bright enough for their spectra to be observed in detail. This spectrometric method is described in Section 3.3.4.

In practice, it is very difficult to measure F. One reason for this is the limited spectral range over which any one flux detector will respond. For example, instruments which can detect radiation at visual wavelengths are not suitable for detecting infrared light, and the designs of the telescopes themselves need to be completely different if we wish to detect X-rays or radio waves. We therefore require a great range of detectors, coupled to suitable telescopes, to measure F over the full wavelength range. There is a further difficulty in measuring F if we make measurements from the Earth's surface: the Earth's atmosphere is not perfectly transparent to any wavelengths, and is opaque to many, as you saw in Figure 1.38.

The best we can do is to make measurements over the more transparent wavebands, and apply corrections to obtain the flux density values that we would have obtained at the top of the atmosphere.

QUESTION 3.4

List the more transparent wavebands in the Earth's atmosphere.

We can avoid these difficulties if we can make do with flux density measurements over limited wavelength ranges. We can indeed do so: moreover, a single waveband suffices. Let's illustrate the approach by selecting one often-used waveband – the V band (or V channel). Any V-band detector has the well-defined spectral response shown in Figure 3.28: it approximates the spectral response of human vision, hence the label 'V', for visual. All stars emit a significant fraction of their luminosity in the V band, and the Earth's atmosphere can be very transparent at these wavelengths. Thus we can readily obtain the flux density F_V, where the subscript V indicates that the flux density has been measured with a detector that has the spectral response of the V band.

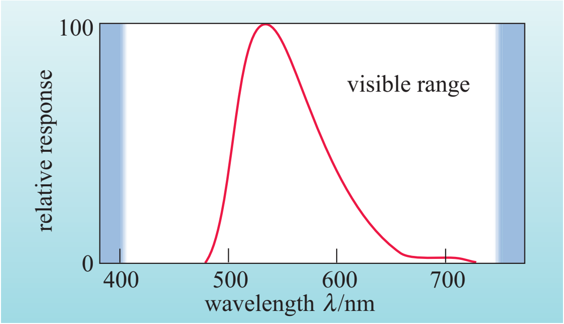

Figure 3.28 The spectral response of a V-band detector.

To obtain the luminosity in the V band, we make use of Equation 3.11, which applies not only to the whole output of a star, but also to any part of it. In the particular case of the V band, Equation 3.11 becomes

$$L_V = (4\pi d^2)F_V \tag{3.13}$$

and Equation 3.12 becomes

$$d = [L_V/(4\pi F_V)]^{1/2} \tag{3.14}$$

L_V is therefore a measure of the total power output of the star in the V waveband. We can then estimate the star's total luminosity L if we know the star's surface temperature. This is because the ratio of power output in the V band to the total power output is uniquely defined by the temperature for a black body, and stellar spectra are similar to black bodies (see Figure 3.18). For the Sun, $L_V = 4.44 \times 10^{25}$ W: you can check that this is reasonable by comparing Figure 3.28 with the 5770 K curve in Figure 3.18b, remembering that $L_\odot = 3.84 \times 10^{26}$ W.

Magnitudes

If you were to look up a table of stellar properties, you would not find F_V and L_V, but closely related quantities called, respectively, apparent and absolute visual magnitudes. Why should astronomers not simply use F_V? It is part of the burden of

history, dating back to antiquity, and particularly to the Greek astronomer Hipparchus. He classified the stars according to their apparent visual brightness in six groups or *magnitudes*, the brightest stars being of first magnitude, the faintest of sixth magnitude, the ones between being placed according to their brightness into an appropriate intermediate magnitude. European astronomers used and refined this system right through to the appearance of detectors that could measure F_V. Such measurements showed that a first magnitude star is about 100 times brighter than a sixth magnitude star. Since the human perception of brightness, as measured by the eye, is logarithmic in nature (i.e. equal steps of perceived brightness correspond to equal *ratios* of flux density – see Box 3.2 if you are not familiar with logarithms) the magnitude scale is a logarithmic scale.

BOX 3.2 LOGARITHMS

A logarithm (or 'log' as it is usually abbreviated) is a mathematical formulation which was widely used to perform long calculations before the advent of the electronic calculator. We will not discuss the theory of logs here but merely their properties, which are relevant to the magnitude system. If p is the log of the number x, this is written

$$p = \log_{10} x$$

and means 10 to the power of p is equal to x, i.e.

$$x = 10^p$$

The number 10 is called the 'base' and p is correctly defined as 'the logarithm to the base 10 of x', but is often shortened to 'p is the log of x' and written

$$p = \log x$$

You can determine the value of p, for any number, x, using the 'log' button on a calculator. Conversely, if you have a value of p, then x can be determined using the 10^x button (sometimes labelled 'antilog'). In the past, books of tabulated conversions, 'log tables', were required.

Let's look at some values: if $x = 10$ (i.e. 10^1) then $p = 1$; if $x = 100$ (i.e. 10^2) then $p = 2$; if $x = 1000$ (i.e. 10^3) then $p = 3$; and so on.

Note that as each value of the number, x, increases by a *factor* of 10, the value of p increases by the *addition* of 1.

You can see that p can have negative values: If $x = 1$, (i.e. 10^0) then $p = 0$, if $x = 0.1$ (i.e. 10^{-1}) then $p = -1$, if $x = 0.01$ (i.e. 10^{-2}) then $p = -2$. Note that no value of p exists for $x = 0$ or a negative value of x.

■ If the log of 8 is 0.9, what is the log of (a) 80, (b) 8000 000, (c) 0.8?

❑ You can use your calculator but it is not necessary:

(a) 80 is 10 times larger than 8 so its log is $0.9 + 1 = 1.9$.

(b) 8000 000 is 10^6 times bigger than 8 (i.e. $10 \times 10 \times 10 \times 10 \times 10 \times 10$) so its log is $0.9 + 6 = 6.9$.

(c) 0.8 is 10 times smaller than 8 so its log is $0.9 - 1 = -0.1$.

This illustrates a very important property of logs. The range of values of p is small compared with the range of values of x. If you wish to display data with a very wide range of values on a graph, the use of logarithmic scales is recommended. An example is the horizontal axis of Figure 3.29.

QUESTION 3.5

What are the values of the logs of: (a) 5, (b) 50, (c) 5000 000, (d) 0.5, (e) 5×10^{-7}?

When performing calculations using logs, you should not confuse logs to the base 10 with *natural* logarithms, which have a different 'base'. In this case $x = e^q$ where the 'base', e, is a number ≈2.718 … and the natural log is written $q = \ln x$ (but is sometimes confusingly written $q = \log x$). On calculators the natural log is usually obtained from the button marked 'ln'.

Each difference of 1 magnitude corresponds approximately to a ratio of 2.5 in brightness. This was precisely quantified so that a difference of 5 magnitudes was set to be precisely a factor of 100. This meant that a difference of 1 magnitude corresponds to a factor of 2.512 in brightness since $(2.512 \times 2.512 \times 2.512 \times 2.512 \times 2.512) = (2.512)^5 = 100$. The difference in magnitudes of two stars may be expressed in the form of an equation

$$(m_1 - m_2) = -2.5 \log(b_1/b_2) \quad (3.15)$$

where m_1 and m_2 are the apparent magnitudes of stars 1 and 2, and b_1 and b_2 are the apparent brightnesses of stars 1 and 2. The brightnesses are defined by the flux density F, although they can, in this equation, be measured in any units (e.g. output signal from a measuring device) because the units cancel in the ratio b_1/b_2.

If the flux densities of the stars are measured in the V band, then b_1 and b_2 correspond to measurements of F_V for the two stars. m_1 and m_2 are then called **apparent visual magnitudes** and denoted m_V or simply V.

Note that the magnitude scale is 'upside down'. The greater the apparent magnitude, the fainter the star. Figure 3.29 shows the relationship between F_V and V. The use of telescopes meant that objects fainter than magnitude 6 became observable and so the scale was extended to larger values of V. Also, a few stars were found to be brighter than first magnitude so negative values of V are possible. It is apparent from Figure 3.29 that the magnitude scale is rather a good method for quoting stellar brightnesses. While F_V varies from 10^{-8} W m^{-2} for Sirius, the brightest star in the night sky, to 10^{-20} W m^{-2} for the faintest detectable objects, the V magnitude ranges only from −1.5 to about 27.

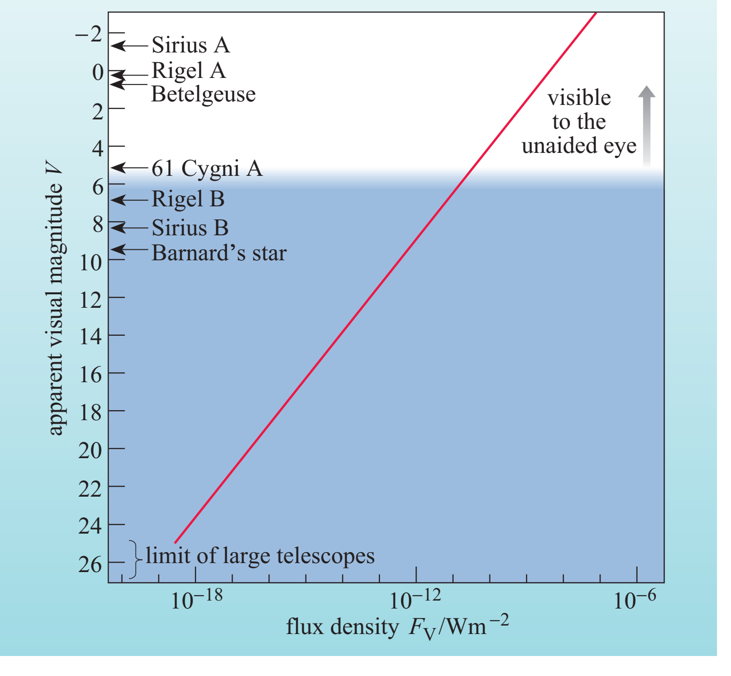

Figure 3.29 The relationship between F_V, the flux density in the V band, and apparent visual magnitude V. Approximate values of V are indicated for a number of objects. Note the log scale for F_V.

EXAMPLE 3.1

Two stars are observed with a telescope and photometer (an instrument which is sensitive to visible light and produces a signal proportional to the star's flux density). Star X, which has apparent magnitude 5.5, produces a signal 40 times stronger than star Y. What is the apparent magnitude of star Y?

SOLUTION

Using Equation 3.15, set star 1 as star X and star 2 as star Y

$$(m_{\text{star X}} - m_{\text{star Y}}) = -2.5\log(b_{\text{star X}}/b_{\text{star Y}})$$

The ratio $b_{\text{star X}}/b_{\text{star Y}} = 40/1 = 40$, so

$$(m_{\text{star X}} - m_{\text{star Y}}) = -2.5\log(40) = -2.5 \times 1.6 = -4.0$$

The apparent magnitude of star X, $m_{\text{star X}} = 5.5$, so

$$(5.5 - m_{\text{star Y}}) = -4.0$$

$$-\,m_{\text{star Y}} = -4.0 - 5.5$$

$$m_{\text{star Y}} = 9.5$$

So the apparent magnitude of the star Y is 9.5.

QUESTION 3.6

The closest star system to the Sun, α Centauri, is a triple system, with stars denoted A, B and C. The brightest, α Centauri A, has a flux density approximately 2.5 times larger than α Centauri B and 25 000 times larger than α Centauri C (also known as Proxima Centauri). If the apparent visual magnitude of α Centauri A is 0, what are the apparent visual magnitudes of α Centauri B and C?

Appendix A4 provides a list of the brightest stars visible from the Earth (i.e. those with the lowest values of apparent magnitude).

As we have seen, the value of F_V, and therefore of V, is not an intrinsic property of a star, but depends on the distance to the star (and on the effects of any intervening interstellar material). By contrast, L_V is an intrinsic property. Likewise, the **absolute visual magnitude**, M_V, is also an intrinsic property: it is the value of V that would be obtained if the star was placed at a standard distance of 10 pc (in the absence of any interstellar matter).

In the same way that the observed flux density depends on the luminosity and distance of a star, the apparent magnitude depends on the absolute magnitude and distance. The absolute magnitude M is given by

$$M = m - 5\log d + 5 \qquad (3.16)$$

where m is the apparent magnitude and d is the distance in parsecs. Most stars have absolute visual magnitudes within the range $-6 < M_V < 16$. (Remember a star with $M_V = -6$ is much brighter than a star with $M_V = 16$. Using Equation 3.15 shows that it is over 6×10^8 times more luminous.) The absolute visual magnitude of the Sun is 4.8 indicating that it is a very average star.

Magnitudes are also used to express colours of stars. Flux density can be measured using detectors and filters which are sensitive to different wavelength ranges. The best known is the **'UBV' system**; in addition to the V (visual) passband (centred at wavelength 550 nm), U ('Ultraviolet' centred at wavelength 365 nm) and B ('Blue', centred at wavelength 440 nm) are defined. Many other passbands extending into the red (R) and infrared (I, J, H, K …) have been defined but here we will concentrate only on B and V. As you saw in Section 3.3.2, the photometric method employs measurements at two different wavelengths to define the temperature of a star. The *difference* in magnitudes measured in the B and V bands, $B - V$, is called the **colour index** and is a measure of the *ratio* of flux densities in the B and V bands (see Equation 3.15) and is therefore an indicator of temperature. $B - V$ values typically range from –0.4 for very hot stars (spectral type O) to +1.5 or more for very cool stars (spectral type M).

QUESTION 3.7

The brightest stars in the southern constellation Centaurus, α Centauri and β Centauri have similar apparent visual magnitudes, $V = 0.01$ and 0.61, and lie at distances of 1.35 and 161 pc respectively. What are their absolute visual magnitudes? How do the luminosities of these stars compare with that of the Sun?

3.3.4 The power of spectroscopy – luminosities, radii and distances

In the early years of the 20th century, astronomers compared the spectra of stars with similar temperatures but different luminosities, the luminosities having been obtained non-spectrometrically. They found that, *at a given temperature*, the more luminous the star the narrower its spectral absorption lines, and the stronger the absorption lines due to certain ionized atoms.

An absorption line is *not infinitely thin*, but covers a narrow range of wavelengths. You should remember the difference between the *strength* and the *width* of an absorption line (Figure 3.21). In Figure 3.30 the two lines have the same strengths (areas), but different widths. Real examples are shown on the right.

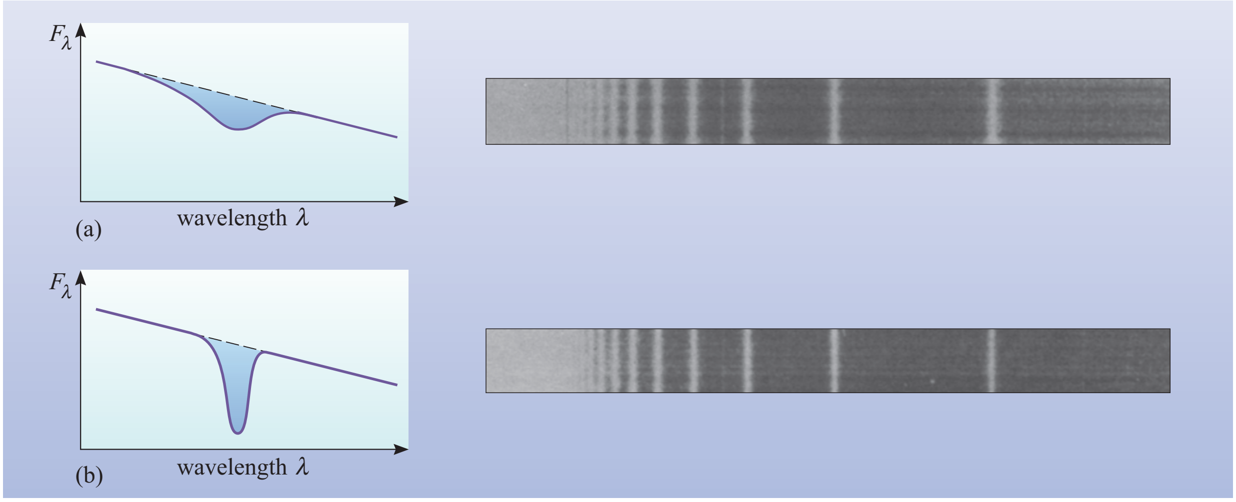

Figure 3.30 The effect of stellar luminosity on spectral line width showing a graphical representation and a real example. (a) A small star and (b) a large star at the same temperature; note that these are *negatives* of photographs, so that the absorption lines appear bright on a dark background. (Spectra from Abt *et al*., 1968)

The reason for these effects of luminosity on linewidth, and on the strengths of certain ion lines, is the difference in conditions present in the atmospheres of the stars where the lines originate. The outer layers (where spectral ions are formed) of a large star has a density lower than those of a small star of the same temperature because the mass of these layers is spread over a larger volume. Atoms and ions are therefore spread over a larger volume and collisions between them are relatively infrequent. In small stars however, the density and pressure are higher and the atoms and ions collide and interact with each other. These interactions cause distortions in the energy levels of the atoms and therefore cause small random differences in the energy change (and hence wavelength) when a photon is absorbed. The spectral lines are therefore broader for small (i.e. lower luminosity) stars. This process is called **pressure broadening**. In addition, the probability of an ion interacting with an electron and recombining is higher in a small star so the spectral lines produced by that ion are weaker than for a large star, because there are fewer ions present to absorb photons.

As soon as astronomers had correlated luminosity with such spectral features, then for stars of unknown luminosity these spectral features could be used to determine their luminosity. For example, the widths of hydrogen lines are used to define luminosity for spectral types B to F, whereas the relative strengths of the ionized strontium (SrII) and neutral iron (FeI) lines are used as a luminosity indicator for spectral classes F to G. This method does not provide precise luminosities, but is used to define *luminosity classes* which distinguish different groups of stars. You will learn more about luminosity classes in Section 4.2.2.

Using this spectrometric method, many stellar luminosities have been measured, and they cover an *enormous* range, from less than 10^{-4} times that of the Sun, to over 10^6 times the solar value. Thus, the Sun is a fairly average star. Most of the stars that we see with the unaided eye are more luminous than the Sun: for example, Rigel A and Betelgeuse are over 10^5 times as luminous.

This is not because bright stars are common – in fact they are not – but because if dim stars, such as the Sun, are at distances beyond about 20 pc, they are visible only through a telescope!

Note that the effect of luminosity on the strength of lines of some ionized atoms means that spectral classification can be influenced by luminosity as well as temperature: remember that spectral classification depends on the strengths of various spectral lines, including lines from ionized atoms. Fortunately, over a wide range of luminosities, the effect of luminosity is a good deal less than the effect of temperature. Moreover, the effect of luminosity on spectral type can be allowed for. Thus the evaluation of temperature on the basis of spectral line strengths is not undermined by the effect of luminosity.

EXAMPLE 3.2

You have seen that the temperature and luminosity of a star can both be obtained spectrometrically. This gives us a new way of obtaining stellar radius.

(a) Explain what this new way is and derive an expression for the radius in units of solar radius.

(b) Aldebaran B is the fainter of the two stars in the binary system Aldebaran (α Tau; the brighter is Aldebaran A). For Aldebaran B, $T = 3400$ K and $L = 0.06L_{\odot}$ where $L_{\odot}$ is the solar luminosity. Calculate its radius giving your answer in solar radii.

SOLUTION

(a) Stellar radius can be obtained from Equation 3.9,

$$L \approx 4\pi R^2 \sigma T^4$$

Since both L and T are defined spectroscopically the only unknown quantity is R. Rearranging Equation 3.9 gives

$$R^2 \approx L/(4\pi\sigma T^4)$$

so $$R \approx [L/(4\pi\sigma T^4)]^{1/2}$$

A convenient form of this equation is obtained by putting the luminosity L, temperature T and radius R in solar units. Equation 3.9 can be expressed for the Sun,

$$L_\odot \approx 4\pi R_\odot^2 \sigma T_\odot^4$$

Dividing the standard form of Equation 3.9 by the solar version (i.e. dividing the left-hand side by $L_\odot$, and the right-hand side by the equivalent quantity $4\pi R_\odot^2 \sigma T_\odot^4$). We thus get

$$L/L_\odot \approx (R/R_\odot)^2 \times (T/T_\odot)^4$$

Which can then be rearranged to give

$$R/R_\odot \approx (L/L_\odot)^{1/2} \times (T_\odot/T)^2$$

This equation is useful because we can use it to compare properties of stars in solar units.

(b) For Aldebaran B

$$R/R_\odot \approx (0.06)^{1/2} \times (5770/3400)^2 \approx 0.7$$

and so $R \approx 0.7R_\odot$.

The radius of Aldebaran B is 0.7 times the radius of the Sun.

Not only do we now have a new way of obtaining radius, we also have a new, and very important way of obtaining distance. We can obtain L and T spectrometrically and hence use Equation 3.14 to obtain the distance to a star from the observed flux density F_V. This is called the method of **spectroscopic parallax** for obtaining stellar distances, a perverse name, given that it has nothing to do with the method of trigonometric parallax outlined earlier!

The method is subject to error, in that F_V can be reduced by interstellar matter. Even in the absence of such a reduction, or if a correction is applied for it, the distance is rather uncertain, because the luminosity obtained from spectral lines is itself uncertain. However, the method can provide reasonable estimates of the distances to very bright stars well beyond those that we can obtain at present from trigonometric parallax, and it is therefore an important method of distance determination.

QUESTION 3.8

The star ι^1 Sco (Greek iota with the '1' distinguishing it from a nearby star ι^2) has $F_V = 4.4 \times 10^{-10}\ \mathrm{W\,m^{-2}}$ (after correction for reduction by interstellar matter) and $L_V = 6.1 \times 10^{30}$ W. Calculate the distance to this star, expressing your answer in parsecs. To the eye, ι^1 Sco does not seem very bright. Given that its apparent brightness is not much reduced by interstellar matter, is it in fact a bright star at a large distance?

3.3.5 Are these properties constant?

Most stars appear to have constant properties. Over long timescales (millions to billions of years) stars evolve and their characteristics change but this process cannot generally be observed directly. Stellar evolution is the subject of Chapters 5 to 9.

However, some stars do change in brightness on measurable timescales from seconds to years and these are collectively called **variable stars**.

Extrinsic variables change as a result of geometrical effects. The majority of this type are eclipsing binaries (which you have already met in Section 3.2.3). **Intrinsic variables** change in brightness as a result of physical changes in the stars themselves.

Regular variables exhibit variations that are regular in time, and so the variations have fairly well defined periods. **Irregular variables**, as their name implies, exhibit variations that are irregular in time; some types change continuously, others stay virtually constant for long periods and then brighten (or dim) suddenly.

There are many different types of variable stars with a wide range of properties, some of which you will meet as we discover more about stellar properties and evolution. Examples of their **light curves** (i.e. how the magnitude changes with time) are shown in Figure 3.31.

About two-thirds of all known variable stars are **pulsating variables**. They are intrinsic variables with luminosity, temperature and radius all changing with time. **Cepheids** ('sefeeds') are named after δ Cephei, the first to be discovered, in 1784, by the English astronomer John Goodricke (1764–1786) and they are one of the most important classes of pulsating variables. They are regular variables with periods of anything from about a day to about 100 days, and their luminosity changes can be up to a factor of 10. Cepheids help us to understand some of the processes that drive evolution at certain stages in a star's life (Section 7.3) and provide us with yet another way to measure distance.

Figure 3.31b shows a light curve of a typical Cepheid variable star. The possibility of using Cepheids for measuring distance was discovered by accident by the American astronomer Henrietta Leavitt (Figure 3.32) from searches for Cepheids in the Small Magellanic Cloud (SMC). The Small and Large Magellanic Clouds are companion galaxies to our own (although this was not known at the time), and visible to the unaided eye from the Southern Hemisphere as apparently disconnected pieces of the Milky Way. She discovered that the longest periods corresponded to the Cepheids with the brightest apparent magnitudes. However, since the Small

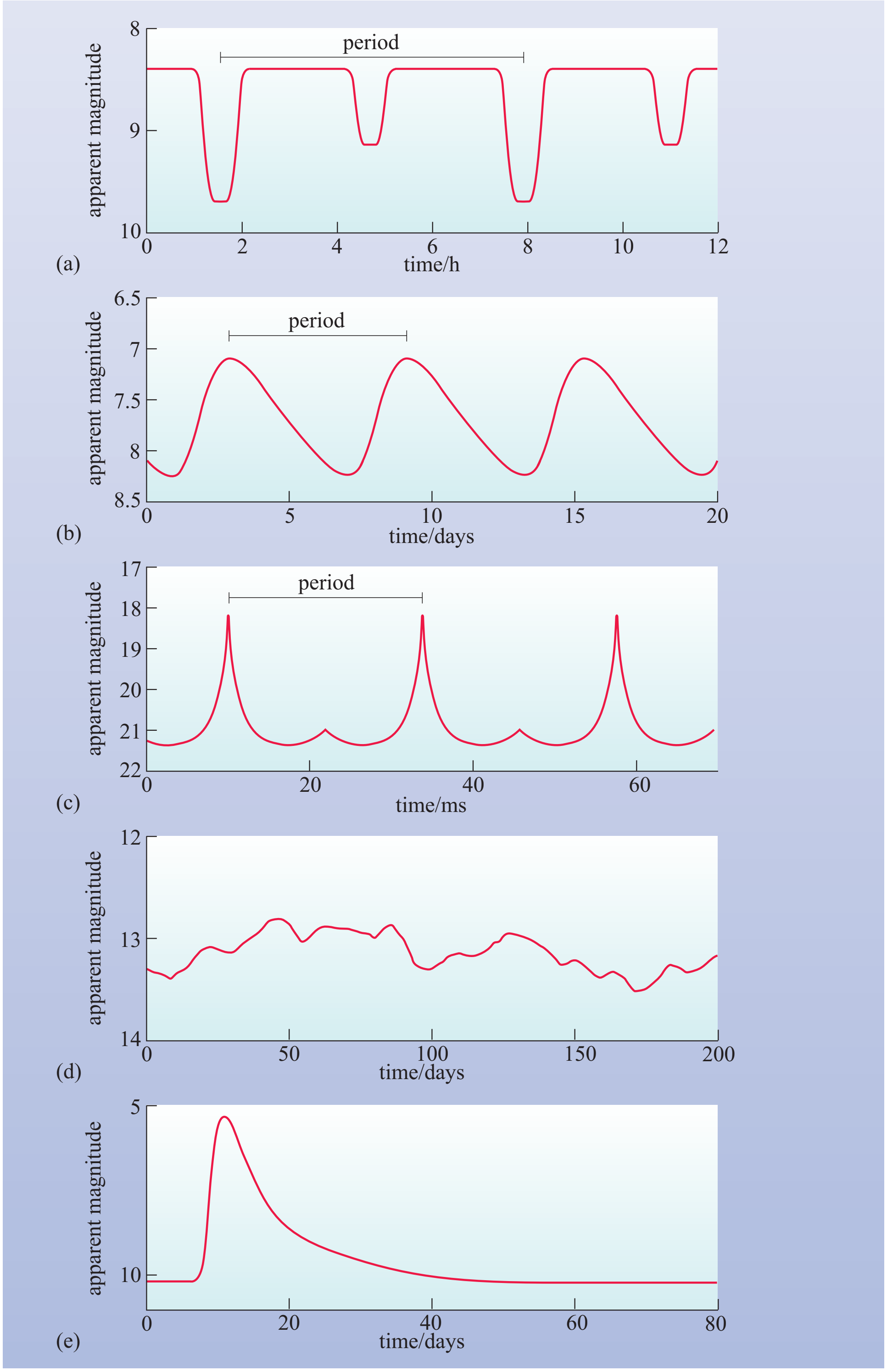

Figure 3.31 Light curves of examples of different types of variable star: (a) eclipsing binary, (b) Cepheid variable, (c) Pulsar – discussed in Chapter 9, (d) T Tauri star, (e) Nova.

Magellanic Cloud is so far away, all the stars could be assumed to be at approximately the same distance. There is therefore also a relationship between period and *absolute* magnitude (see Equation 3.16) and hence between period and luminosity. Once actual absolute magnitudes were determined from independently

HENRIETTA LEAVITT (1868–1921)

Henrietta Leavitt (Figure 3.32) developed an interest in astronomy while an undergraduate at Radcliffe College (now part of Harvard University). She graduated in 1892, and like her future colleague Annie Jump Cannon (Figure 3.24) became partially deaf following illness in the years immediately after graduation. From 1895–1900 she was a volunteer at the Harvard College Observatory, becoming a staff member in 1900, where she worked until her early death from cancer. Best known for her work on variable stars she identified over 2400, doubling the number known in her day. Her discovery of the Cepheid period–luminosity relationship gave the first means of measuring extragalactic distances and led to the recognition that the Magellanic Clouds are actually two companion galaxies to the Milky Way at (for that time) unprecedented distances from the Sun.

Figure 3.32 Henrietta Leavitt. (Royal Astronomical Society)

measured distances for some Cepheids, the **period–luminosity relationship** shown in Figure 3.33 was derived. This simple diagram provides an extremely powerful method for determining distances, since Cepheids are sufficiently luminous to be identifiable anywhere in our Galaxy and in other nearby galaxies. As soon as a Cepheid is identified from its light curve, the absolute magnitude can be deduced from the period and the distance derived using Equation 3.16.

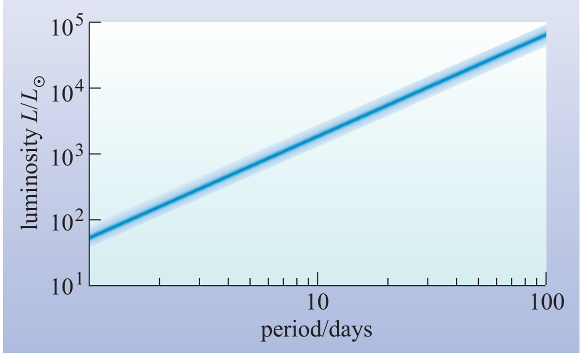

Figure 3.33 The period–luminosity relationship for Cepheid variable stars.

More spectacular irregular variables are the novae, a name that means 'new stars'. A **nova** is a star that brightens by up to 10 000 times (10 magnitudes) in a few days, followed by a slower decline to about its original luminosity – the star is not destroyed. For some novae, only one outburst has been observed. Other novae are known to repeat the performance, though irregularly, with the average interval between outbursts varying from tens of days for some stars to tens of years for others. You will learn more about the reasons for these outbursts in Section 9.5.2.

Supernovae, (e.g. Tycho's new star referred to Section 3.1) exhibit even greater outbursts in luminosity of up to 100 million times (20 magnitudes). However, these are all single events which effectively end the life of the star. The different types of supernovae are discussed in Chapter 8.

There are many other types of irregular variable, but one type that is of particular importance to the study of stellar evolution is the **T Tauri stars** ('tee tory'), named after one star of this kind. These stars exhibit variations in luminosity by factors of two or three over intervals of the order of a few days. T Tauri stars are discussed in Sections 4.2.3 and 5.3.4.

3.3.6 What are the stars made of?

The familiar world around us is dominated by certain chemical elements. Thus, the atmosphere consists mainly of oxygen and nitrogen, rocks and soil consist mainly of compounds of silicon and oxygen with metals such as calcium, aluminium, magnesium and iron, and the oceans consist almost entirely of water, a compound of two atoms of hydrogen with one atom of oxygen. The Earth as a whole is made up largely of the elements iron, silicon, oxygen and magnesium, all but iron being present mostly in compounds.

Stars represent a much larger sample of the cosmos than that provided by the Earth, and they are very different from the Earth in their composition. You have already seen in Section 2.2.3, that although stars do contain the sort of elements that dominate the Earth, stellar compositions are dominated by two elements that are but *minor* constituents of our planet, namely hydrogen and helium. Before we look more closely at stellar composition, let's consider briefly how it is measured.

Stellar compositions are obtained from studies of the spectral absorption lines in stellar atmospheres. If we can see the lines of a certain element then we can conclude that the element is present. But how much of it is there? The strength of a single absorption line is *not* a good guide.

■ Why not?

❑ Line strength is sensitive to temperature – see Section 3.3.2.

However, we can obtain the amount of an element present by comparing the absorption line strengths of different lines from the element, and it helps if lines are observed not only from the neutral element, but also from the ionized element. The great range of wavelengths covered by the various elements, particularly when we include their ionized forms, led to an enormous flood of data in the second half of the 20th century with the advent of infrared and ultraviolet astronomy. Thus, today we have very extensive knowledge of the compositions of the atmospheres of many stars.

QUESTION 3.9

Use is made of more than one absorption line in obtaining *temperature*. In what way does this procedure differ from the one to obtain *composition*, where more than one line is again used?

We shall not go into further details of how stellar compositions are obtained, except to note that spectrometry is used to obtain composition, temperature and luminosity in not quite such a separate manner as implied above. Instead, a model is constructed of a stellar atmosphere, and the composition, temperature and luminosity in the model are adjusted simultaneously until the model reproduces the great range of observed line strengths and line shapes of all the various elements.

Stellar atmospheres are largely made up of hydrogen, with remarkably small variations from star to star. For a star of average composition, like the Sun, 73% of the mass of the atmosphere consists of hydrogen. This is the lightest element, and so its dominance is even more impressive when expressed in terms of the percentage of nuclei, rather than of mass: 92% of the nuclei are hydrogen. Next comes helium, the next lightest element, at 25% by mass and 7.8% by number. If

you add the hydrogen and helium figures together you will see that there is not a whole lot left for the 90 or so ‘heavy’ elements (atomic number more than 2) – only about 2% by mass and 0.2% by number!

It is important to realize that these measurements are for stellar atmospheres, and so the question arises of whether they are typical for whole stars. Here we have to appeal to stellar modelling, where the model has to reproduce the observed properties: the modelling procedure is similar to that for the Sun, outlined in Section 2.2.2. We find that, except for the cores of stars where considerable nuclear fusion has presumably occurred, stellar interiors are indeed dominated by hydrogen and helium, the above values being typical for most stars. Among the heavy elements considerable variations do occur, though among many stars there are still remarkable similarities.

A standard composition, the *solar system abundance*, has been defined, based on the Sun outside its core and on the composition of a particular class of meteorites (small bodies that reach the Earth’s surface from interplanetary space) – carbonaceous chondrites, which are thought to have changed little since the origin of the Solar System. The standard composition is thus that of the material from which the Solar System formed. (Figure 3.34 shows the broad features of this composition, and the full details are given in Appendix A5.) Historically it has been called, rather grandly, the *cosmic relative abundance* of the elements. However, as you will discover in later chapters, this concept is flawed since the detailed abundances in stars and different regions of the Galaxy vary due to the continual processing of the elements within stars and during their death throes.

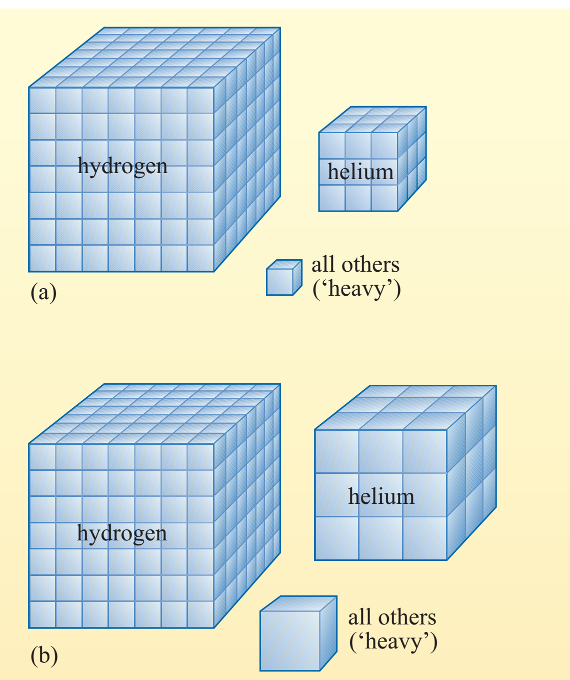

Figure 3.34 The large cubes represent the relative abundances of the elements in the solar neighbourhood (a) by relative numbers of atomic nuclei and (b) by relative mass. The *number* of particles in each small cube is the same.

QUESTION 3.10

From Appendix A5, what are the ten most abundant elements by number of nuclei, and by mass?

3.3.7 Measuring stellar masses

That stellar masses *can* be measured probably comes as no surprise to you in the light of all the measured stellar properties that you have already met. So, how is it done? The basis of the method is to observe how a star moves when a force is applied to it: this is a direct application of Newton's second law of motion, which states that the force applied to a body is equal to its mass times its acceleration. Therefore we must observe a star being accelerated.

Any star is accelerated by any other mass in the Universe through gravitational attraction, but the effects are only large enough to measure if the other mass is large, and relatively close to the star. Of particular importance therefore are systems in which just two stars are sufficiently close together to be in orbit around each other – these binary systems were introduced in Section 3.2.3.

Stellar masses are most readily obtained from those binary systems in which both stars can be *seen* to be orbiting each other, i.e. visual binary systems. The procedure, described below, is of wide applicability: for example, it can be used to obtain planetary and satellite masses in the Solar System.

Obtaining the sum of masses, $M + m$

Note that the symbols M and m have also been used for apparent magnitudes. This should not cause any confusion as the context and units should indicate which is meant.

Suppose that two stars are in circular orbits around each other, as in Figure 3.35a. This is a view from a point of observation that is not accelerating. One special point in the system itself is also not accelerating, and it lies on the straight line joining the two stars. This is called the **centre of mass** of the system, marked with a cross in Figure 3.35a. It lies at the centre of each of the circular orbits of the two stars, and thus the orbits shown are with respect to this centre of mass. The stars, however, *are* accelerating: their speeds are constant, but their directions of motion are constantly changing. Therefore, there must be a force acting on each of them.

■ What are these forces?

❑ The force on the star of mass m is the gravitational force exerted on it by the star of mass M, and vice versa. (The *speeds* are constant because the force on each star is always perpendicular to its direction of motion.)

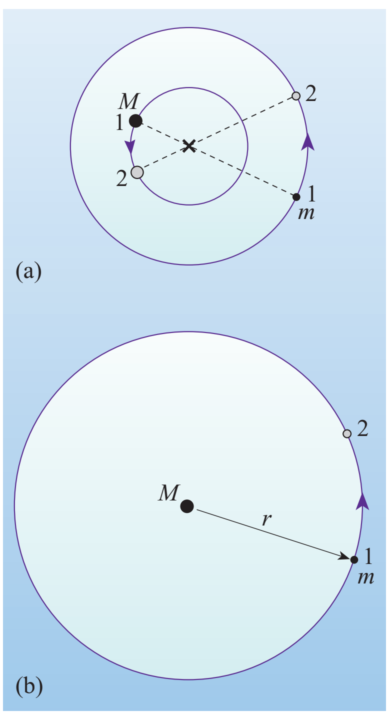

Figure 3.35 Circular orbits in a binary system: (a) relative to the centre of mass (×) of the system; and (b) of m relative to M.

Suppose now that, by some extraordinary feat, we were to sit on M and observe m. It would appear to move around us in an orbit that is called the orbit of m relative to M. If, as here, the orbits with respect to the centre of mass are circular (Figure 3.35a), then this relative orbit is also circular, as shown in Figure 3.35b, with m moving around its relative orbit at constant speed. Furthermore, the orbital periods are also the same from both points of view.

The relative orbit is important because its radius appears in the equation used to describe the gravitational force between the two stars. This force has a magnitude, F, given by Newton's law of gravitation

$$F = GMm/r^2 \qquad (3.17)$$

where G is the gravitational constant (a universal constant), and r is the distance between the centres of the two stars – *this is also the radius of the relative orbit*.

Suppose that the orbital period of m is P. How do we expect P to depend on M, m and r? First, if r is kept fixed, then from Equation 3.17 we see that (reasonably

enough) as M or m increases, F increases. Thus there will be a more rapid change of direction, and so P should decrease. Second, if M and m are kept fixed, then from Equation 3.17 we see that (again reasonable) as r increases, F decreases, and moreover the distance around the orbit, $2\pi r$, increases. Therefore, P should increase. Summarizing:

- as M or m increases, we expect P to decrease
- as r increases, we expect P to increase.

These expectations are borne out by the mathematical details, which won't concern us, but which lead to

$$P = 2\pi\left(\frac{r^3}{G(M+m)}\right)^{1/2} \tag{3.18}$$

We have thus reached the point where, for a circular orbit, if we measure P and r, then we can obtain the sum of the two masses.

QUESTION 3.11

Rearrange Equation 3.18 to obtain the explicit form for $M + m$ (i.e. $M + m = \ldots$).

Elliptical orbits

A circular orbit is rather a special case. In general, the orbit of m relative to M is an ellipse, with M at one of two special points called the **foci** (plural of **focus**) of the ellipse. Various **elliptical orbits** are shown in Figure 3.36. The size of an ellipse is given by the **semimajor axis** a, which is half of the long dimension, and which is the average separation between the two masses. The extent of the departure from circular form is called the **eccentricity** of the ellipse, and is defined as half the distance between the two foci of the ellipse (Figure 3.36) divided by the semimajor axis. In a circle, both foci coincide at the centre, and so the eccentricity of a circle is zero.

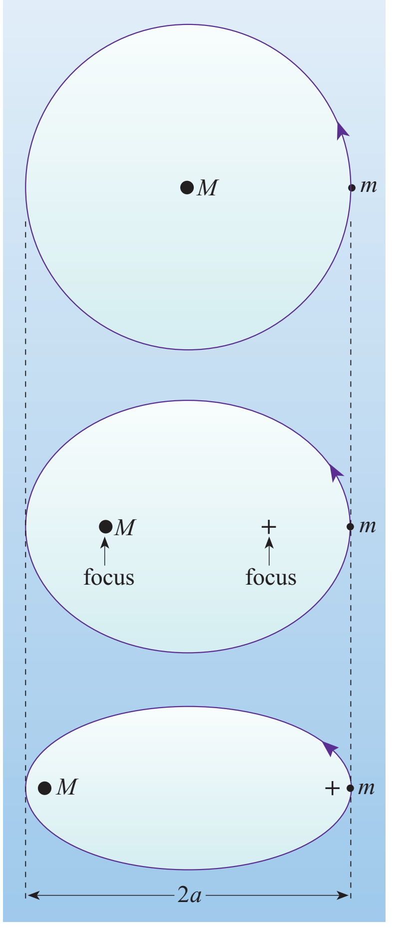

Figure 3.36 A variety of elliptical orbits.

The generalization of Equation 3.18 to elliptical orbits is remarkably straightforward: we simply replace the constant separation r in a circular orbit by the average separation a in an elliptical orbit. Thus

$$P = 2\pi\left(\frac{a^3}{G(M+m)}\right)^{1/2} \tag{3.19}$$

This equation can be rearranged (as in Question 3.11) to give

$$M + m = \frac{4\pi^2 a^3}{GP^2} \tag{3.20}$$

We would have obtained the same result had we considered the orbit of M relative to m, rather than m relative to M.

Thus by measuring P and a we can again get the sum of the two masses, $(M + m)$.

Obtaining M and m separately

To obtain M and m separately we need to identify the position of the centre of mass of the binary system. Recall from Figure 3.35a that this lies on the straight line connecting the stars. We obtain its position on this line by making at least two observations, well separated in time. The centre of mass lies on both straight lines, and so must lie where they intersect: Figure 3.37 illustrates this procedure for the case of Sirius, where the orbits about the centre of mass are shown (elliptical in this case). These orbits are obtained by observing the motion of each star with respect to a fixed frame of reference, which for practical purposes is provided by stars that are much further away than the binary.

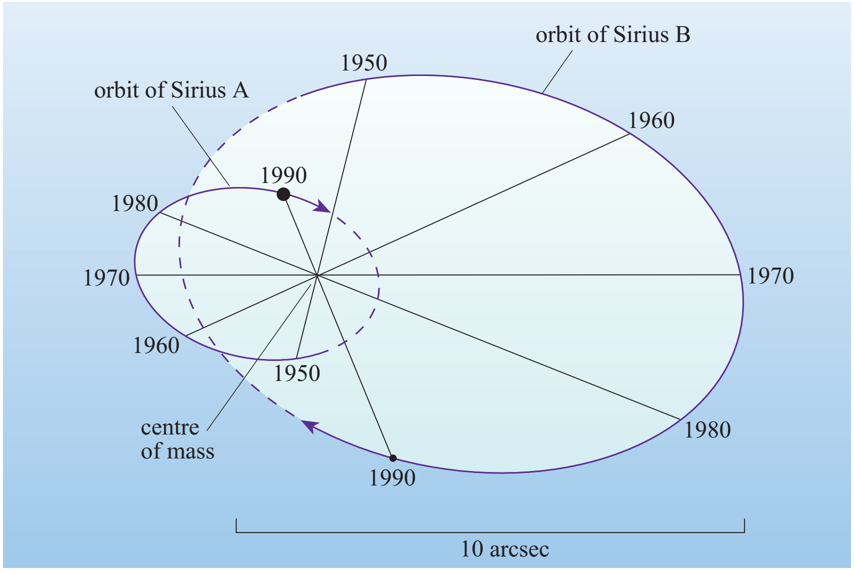

Figure 3.37 Sirius, a binary system, showing the position of the centre of mass at the intersection of lines joining stars A and B at specific times. (Prepared with the assistance of M. A. Seeds)

Consider an instant when the distances of the two stars from the centre of mass are d_m and d_M. It can be shown that the ratio d_m/d_M is constant throughout the orbital motion, and is related to the masses via

$$M/m = d_m/d_M \tag{3.21}$$

This equation is reasonable: if $M \gg m$, then $d_M \ll d_m$; that is, the centre of mass is close to the larger mass, as we might expect.

We have now achieved our goal: from Equations 3.20 and 3.21 we can obtain the masses M and m.

QUESTION 3.12

Suppose that observations have revealed that, for a binary system, $M + m = 12M_\odot$, and $M/m = 3$. Calculate the values of M and m.

Note that Equation 3.21 requires that stars behave as rigid bodies, so that they don't deform each other. In fact, stars are not at all rigid! However, provided that they are small compared with their separation then the deformations are negligible. For all visual binaries, this condition is met. This wide separation also means that the stars do not interfere with each other's evolution.

There are three important observational details to point out when deriving stellar masses from observations of binary stars:

First, it is important to realize that in Figures 3.12 and 3.37, as for any visual binary, we are observing the relative orbit as a projection onto a plane perpendicular to our line of sight, as illustrated in Figure 3.38. We thus observe the true shape of the relative orbit only if the actual relative orbit happens to lie in this plane. Furthermore, the semimajor axis a appears shorter in the projection plane than it really is. Fortunately, the true value of a, for use in Equation 3.19, can be deduced from the observational data: in essence, we are able to 'deproject' the relative orbit,

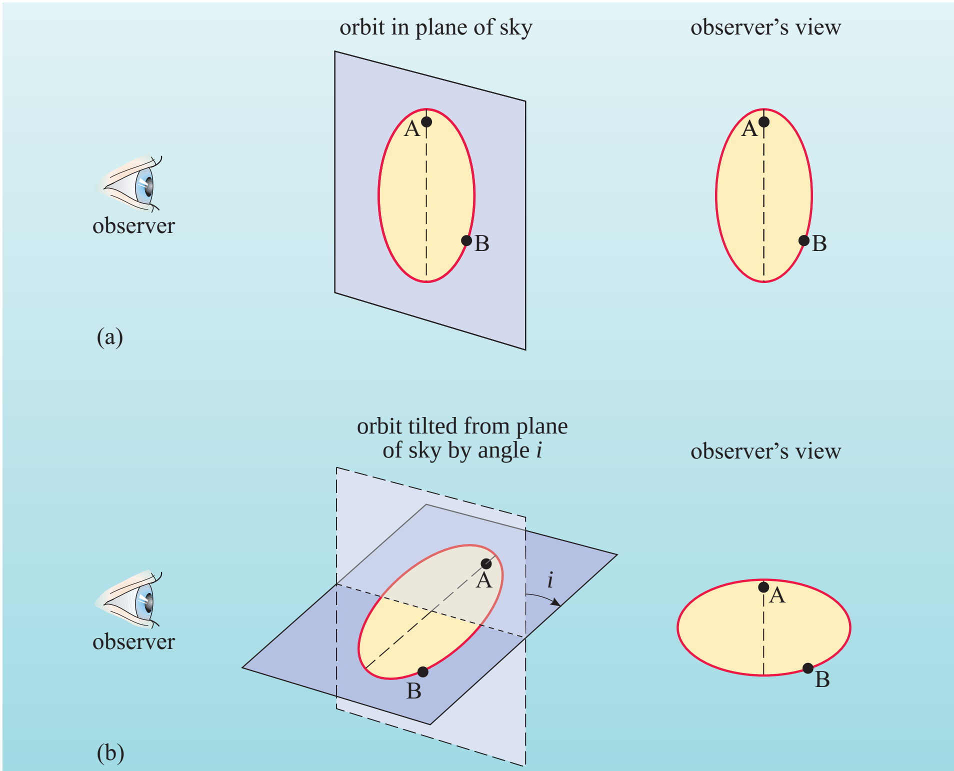

Figure 3.38 The change in appearance of an orbit due to its projection onto the plane of the sky. The dashed line shows the major axis of the orbit. In case (b) the true major axis is not the longest axis when projected onto the plane of the sky.

though the details will not concern us. Note that the ratio d_m/d_M in Equation 3.21 is unaffected by the projection; d_m and d_M are each shortened by the same factor. The orbital period P is also unaffected.

Second, in order to obtain a we also need to know the distance to the binary: what we actually measure is the *angular* separation of the two stars, which gives us the projected value of the semimajor axis, a', using

$$a' = (\alpha/\text{radians}) \times d$$

(obtained by substituting the a', for $2R$ in Equation 3.8). The semimajor axis, a, is then determined from a' and the tilt of the orbital plane. The distance can be obtained by one of the methods that you met earlier in this chapter.

Third, note that the method of obtaining masses from binary systems outlined above requires that both stars be seen as distinct points of light, that is, we must be dealing with a *visual* binary system. Unfortunately, most binary systems are not of the visual kind. In some cases, we can detect the radiation from only one of the two stars. However, the second star makes its presence felt through its effect on the proper motion of the other star: the proper motion is not a smooth line but displays orbital wiggles (see Figure 3.39). These pairs of stars are called astrometric binaries. Indeed, before Alvan Clark saw Sirius B in 1862, its existence had been inferred by Bessel in 1844 from such wiggles in the proper motion of Sirius A.

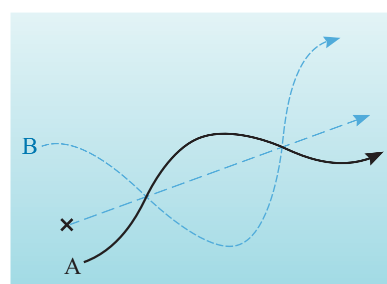

Figure 3.39 The apparent motion of an astrometric binary star. The solid track is the proper motion of a visible star against the background of distant stars. The invisible companion, B, follows the curved dashed line and the centre of mass (marked with a cross), defines the proper motion of the binary star system as a whole.

It should be apparent from the discussion of binary stars in Section 3.2.3 that only nearby or well separated stars can be observed as visual binaries. This limits the number of stars for which we are able to obtain masses by this technique. However, it is possible to derive information on stellar masses from spectroscopic binaries (Section 3.2.3). If we assume the two stars are in circular orbits about the

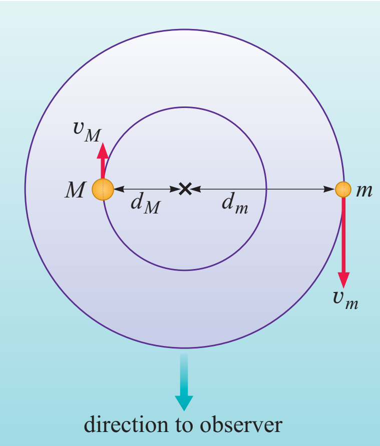

Figure 3.40 Circular orbits in a spectroscopic binary system relative to the centre of mass (×) of the system. The stars are at a position where the maximum separation of their spectral lines will be observed due to the Doppler shift.

centre of mass, with the orbital plane perpendicular to the plane of the sky (i.e. in the line of sight as shown in Figure 3.40), then we can derive the orbital speeds from the maximum Doppler shifts of spectral lines from the two stars (see Figure 3.9). The orbital speed of each star equals the distance around its orbit divided by the time taken for one orbit, i.e. the period P:

$$v_M = 2\pi d_M/P, \quad v_m = 2\pi d_m/P \tag{3.22}$$

Since the ratio of masses is related to the ratio of distances from the centre of mass (Equation 3.21) then from Equation 3.22 we obtain

$$M/m = d_m/d_M = v_m/v_M \tag{3.23}$$

P is obtained from the period of oscillation of the spectral lines so d_M and d_m can be calculated *individually*. The distance between the two stars, r, which in this case is also the semimajor axis, a, is simply (Figure 3.40)

$$r = a = d_M + d_m \tag{3.24}$$

We therefore can use Equations 3.20 and 3.23 to derive the masses of the two stars. Since the identification of spectroscopic binaries does not depend on distance to the binary (it only requires that the stars are bright enough for spectra to be obtained), this method is potentially very powerful. However, in general, the angle between the orbital plane and the plane of the sky is unknown and hence the observed radial velocity oscillations will be smaller than the true orbital speeds and the stellar masses will be underestimated.

- ■ Are there any types of binary star for which you can be sure that the orbital plane is in the line of sight?
- ❑ Yes, eclipsing binary stars (Section 3.2.3) must have their orbital planes very close to the line of sight for one star to pass in front of the other.

When the above techniques are applied, measured stellar masses are found in the range from about $0.08M_\odot$ to about $50M_\odot$. We will examine the reasons for these limits in Chapter 6.

Stellar radii from eclipsing binaries

Not only can eclipsing spectroscopic binaries be used to derive masses, but they also provide an alternative technique for determining stellar radii. Figure 3.41 illustrates the geometry of an eclipsing binary eclipse with the corresponding light curve. The radii of the stars are derived from the time intervals between brightness changes in the light curve. Between time t_1 and t_2 the small star moves a distance of twice its radius, R_S, at a speed v. If the star is in a circular orbit, then v can be derived from the maximum Doppler shift as shown in Figure 3.40. During the eclipse the small star is moving along only a small part of its orbit so we can assume constant speed and therefore the distance travelled = speed × time taken:

$$2R_S = v \times (t_2 - t_1) \tag{3.25}$$

Similarly, between time t_1 and t_4 the small star moves a distance of $R_S + R_L + R_L + R_S$ at speed v. So

$$2R_S + 2R_L = v \times (t_4 - t_1) \tag{3.26}$$

Subtracting Equation 3.25 from Equation 3.26 gives

$$2R_L = v \times (t_4 - t_2) \tag{3.27}$$

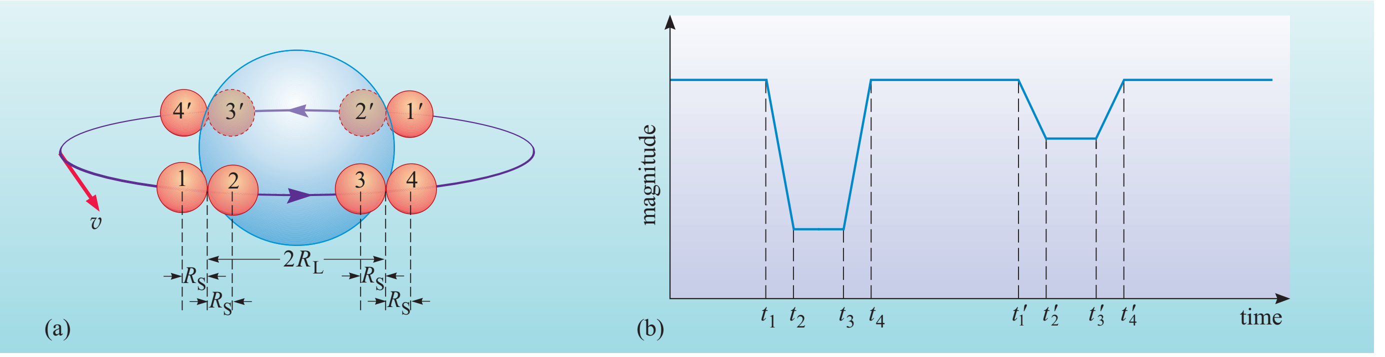

Figure 3.41 (a) Schematic of an eclipsing binary showing the different phases of eclipses. (b) Light curve indicating times of each event during the eclipses. Primary minimum occurs when the hotter star is eclipsed since the light emitted per unit area is greater for the hotter star. The example shows the case when the larger star is the hotter one.

The description of spectroscopic and eclipsing binaries given above assumes circular orbits. In general the orbits are elliptical and the calculations are somewhat more complex.

3.4 Summary of Chapter 3

Stars in space

- Stars move through space with velocities that we split into transverse and radial components. The magnitude of the transverse velocity is given by

$$v_t = d \times (\mu/\text{radians}) \tag{3.1}$$

where μ is the proper motion. The magnitude of the radial velocity (determined from the Doppler shift of spectral lines) is given by

$$v_r = c \times (f - f')/f' \tag{3.3}$$

or

$$v_r = c \times (\lambda' - \lambda)/\lambda \tag{3.4}$$

- The space velocity is given by

$$v = \sqrt{(v_t^2 + v_r^2)} \tag{3.5}$$

The stars are so remote that transverse velocities produce changes to the familiar constellations that would be noticeable to the unaided eye only over intervals of thousands of years.

- Stellar distances were first measured using trigonometric parallax. The distance d is given by

$$d/\text{pc} = \frac{1}{p/\text{arcsec}} \tag{3.7}$$

where p is the stellar parallax. In this equation pc stands for parsec, an important unit of distance in astronomy.

- The distances between the stars are typically a few parsecs, of the order of a million times greater than the average distance from the Earth to the Sun (called the astronomical unit, AU). Trigonometric parallax has yielded stellar distances with useful accuracy to a range of over 100 pc.
- More than half of all stars are found in binary or multiple star systems. Only the closest and most widely separated can actually be distinguished as separate stars. Most are only identified through indirect methods such as changes in their spectra or brightness.
- Two types of star clusters (open clusters and globular clusters), with very different properties are observed in our Galaxy.

Stellar radius

- The stars with the greatest angular diameters as seen from the Earth can have their angular diameters α directly measured. The stellar radius is then given by

  $$R = [(\alpha/2)/\text{radians}] \times d \qquad (3.8)$$

  where d is the distance to the star.

Photospheric temperature

- Photospheric temperature can be obtained by comparing the flux density that we receive from a star in two different wavebands – this is the basis of the photometric method. However, for most stars a better method is to use the strengths of various absorption lines produced by different elements in, and just above, the photosphere. From these lines, a star is assigned to one of the spectral classes O, B, A, F, G, K, M and L and then to one of the subclasses, the spectral types. The average photospheric temperature is then obtained from the relationship between spectral type and temperature. This is the spectrometric method.

Stellar luminosity

- If we know the radius and temperature of a star, then its luminosity (its total power output over all wavelengths) can be obtained from

  $$L \approx 4\pi R^2 \sigma T^4 \qquad (3.9)$$

  The approximation sign arises because a star's spectrum is not quite that of a black body.
- If we know the distance d from a star to the Earth then we can work out the luminosity by measuring the flux density, F

  $$L = (4\pi d^2)F \qquad (3.11)$$

- In general, we don't know the distance of a star, and Equation 3.11 is itself used to determine distance if the luminosity can be derived by an independent method

  $$d = [L/(4\pi F)]^{1/2} \qquad (3.12)$$

- F is almost impossible to measure so a restricted range of wavelengths, defined by a passband, is used. Many stars emit much of their energy in visible light where the Earth's atmosphere is transparent so the V band is commonly used

  $$L_{\mathrm{V}} = (4\pi d^2)F_{\mathrm{V}} \qquad (3.13)$$

Magnitudes

- Magnitudes are a convenient way of expressing the brightness of stars. The magnitude scale is logarithmic and 'upside-down'. The difference in magnitudes of two stars is related to the ratio of their brightnesses

  $$(m_1 - m_2) = -2.5 \log(b_1/b_2) \qquad (3.15)$$

- Apparent magnitudes, m, are a measure of the brightness of a star observed at the Earth. Absolute magnitudes, M, are the apparent magnitude an object would have if moved to a standard distance of 10 pc

  $$M = m - 5 \log d + 5 \qquad (3.16)$$

- Magnitudes measured in the visual waveband are denoted m_{V} (or simply V) and M_{V}. The difference in blue and visual magnitudes, the colour index $B - V$, is an indicator of temperature.

Stellar spectra

- The luminosity of a star can be estimated from the width of its absorption lines, and from the strength of the absorption lines of certain ions.
- We are therefore able to determine both the temperature and luminosity of a star from its spectrum and can therefore derive its radius:

$$R \approx [L/(4\pi\sigma T^4)]^{1/2}$$

- From L and T we can obtain the luminosity L_V in the V waveband. We can readily measure the flux density F_V that we receive from the star in this band, and therefore the distance is given by

$$d = [L_V/(4\pi F_V)]^{1/2} \qquad (3.14)$$

Sometimes it is necessary to correct F_V for the effects of interstellar matter before applying Equation 3.14.

Variable stars

- Some stars change in brightness. These variations may be regular with periods of years or as short as fractions of a second, or irregular.
- Cepheids exhibit a relationship between period and luminosity which allows their distances to be determined by measuring their light curves.

Stellar composition

- The great majority of stars, apart from those in which considerable nuclear fusion has occurred, consist mainly of the two lightest elements, hydrogen and helium. The Sun is fairly typical, with the following composition outside its core:

  hydrogen: 73% by mass, 92% by number of nuclei

  helium: 25% by mass, 7.8% by number of nuclei
- The composition of the Sun, excluding its core, and that of a class of meteorites called carbonaceous chondrites, define a standard composition called the Solar System abundance of the elements.

Stellar masses

- Stellar masses can be obtained from observations of binary systems. This is most directly accomplished using visual binaries, the masses being given by

$$M + m = \frac{4\pi^2 a^3}{GP^2} \qquad (3.20)$$

$$M/m = d_m/d_M \qquad (3.21)$$

- Masses of spectroscopic binaries may be derived using the same equations but only if they are also eclipsing binaries. These allow determination of masses for much more distant stars.
- The light curves and spectra of eclipsing binaries also provide a method of determining the radii of the two stars.

Questions

QUESTION 3.13

The bright star Procyon A ('pro-sigh-on') has a proper motion of 1.259 arcsec yr^{-1}, and a stellar parallax of 0.286 arcsec. Its spectral lines are blue-shifted.

(a) Calculate how many years this star will take to travel an angular distance across the sky equal to the angular diameter of the Moon (0.5°).

(b) Calculate its transverse speed, expressing your answer in km s^{-1}.

(c) Is its radial velocity directed towards us, or away from us?

(d) Why should the large proper motion of this star suggest that it should have a readily measurable parallax?

QUESTION 3.14

(a) Sirius is a binary with stars of very different apparent brightness. Sirius A appears 7600 times brighter than Sirius B. If Sirius A has an apparent visual magnitude $V = -1.46$, what is the apparent visual magnitude of Sirius B?

(b) What can you say about the difference in absolute visual magnitudes of the two stars?

(c) Sirius lies at a distance of 2.64 pc. What is the absolute visual magnitude of Sirius A?

(d) How does its luminosity compare with that of the Sun?

QUESTION 3.15

This question is about the star Rigel A, the prominent bluish-white star in the constellation Orion.

(a) From its spectral type, B8, Rigel A is given a photospheric temperature of 13 000 K. Is this reasonable? How strong are the hydrogen Balmer lines compared with the helium lines, and with the lines of ionized helium and ionized calcium?

(b) Spectroscopic studies lead to an estimate of $1.4 \times 10^5 L_\odot$ for its luminosity. How do its spectral lines differ from those of a far less luminous star of the same temperature?

(c) Calculate its radius, expressing your answer in solar radii.

(d) From its luminosity and photospheric temperature, use Figures 3.18 and 3.28 to make a *rough* estimate of its luminosity, L_V, in the V waveband. Hence estimate its distance, given that $F_V = 3.0 \times 10^{-9}$ W m^{-2}.

(e) Would it have been feasible to obtain its radius from its angular diameter, as measured from the Earth?

QUESTION 3.16

Using Appendix A5, plot a graph of relative elemental abundances in the Solar System by mass versus atomic number Z, for the heavy elements as far as $Z = 30$. Use a linear scale on the abundance axis extending from 0 to a suitable upper limit. (Some of the abundances will be too small to show.) Hence describe briefly the relationship between cosmic relative abundance and Z.

QUESTION 3.17

In the Sirius binary system, the orbital period is 50 years, and the semimajor axis of the relative orbit is 20 AU. From these data, and from Figure 3.37, calculate the masses of Sirius A and Sirius B, expressing your answer in solar masses.